Correction for Linkage Error in Population Size Estimation

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Capture - recapture method

• Estimate the number of fish in a pond

		first	
		yes	no
second	yes	n _{1,1}	n _{o,1}
	no	n _{1,0}	n _{o,o}

		first	
		yes	no
second	yes	14	106
	no	86	??



Capture - recapture method

• Estimate the number of fish in a pond

		first	
		yes	no
second	yes	n _{1,1}	n _{o,1}
	no	n _{1,0}	n _{o,o}



• Assume independency between capture and recapture

OR =
$$\frac{n_{1,1} / n_{0,1}}{n_{1,0} / n_{0,0}} = 1$$
 $\widehat{n_{0,0}} = \frac{n_{1,0} \times n_{0,1}}{n_{1,1}} = \frac{86 \times 106}{14} = 651$



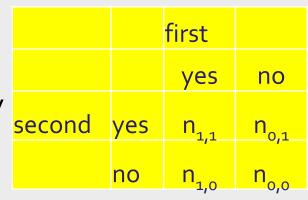
Assumptions

- Instead of samples, you can also use registers
- Assumptions if you use two sources:
 - a. Independency
 - b. Population is closed
 - c. Positive inclusion probability
 - d. Perfect linkage
 - e. No erroneous captures



Assumptions

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Meeting the assumptions

- Independency: three registers used (PR, ER and CSR) and covariates added to the model
- Closed population: ultimo September 2010 for PR and ER, second half of 2010 for CSR.
- Positive probability: only estimation of 15-65 years of age
- Perfect linkage: careful linkage
- Prevent erroneous captures: records which do not belong to the population removed



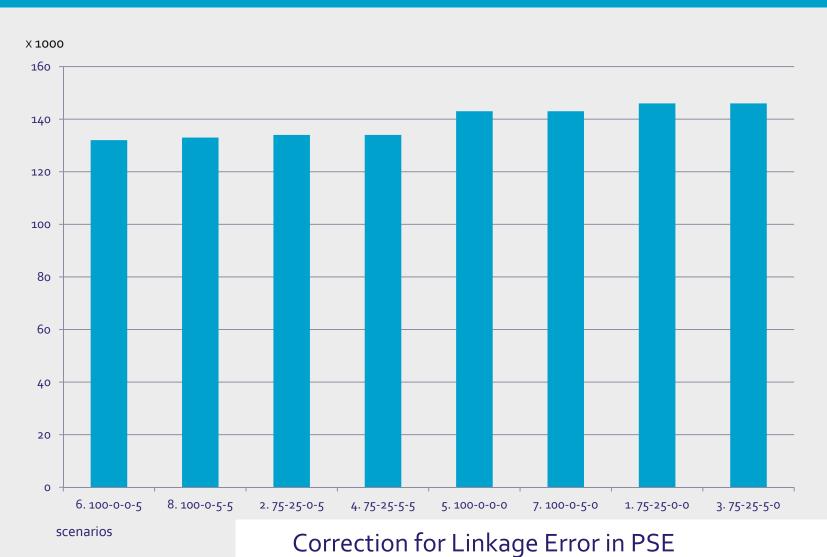
Results

- Estimation for each Nationality group (7 classes).
- With covariates Age (4 classes), Sex (2 classes), Residence duration (2 classes).
- Find the most parsimonious log-linear model that fits the data well with the Bayesian Information Criterion (BIC).

Usual residents not in PR: 470,000 (=5,2%).



Results with the traditional procedure





Probabilistic linkage

Probabilistic linkage:

- Generate all possible pairs
- Large datasets: blocking
- Compute (within blocks) weights for each pair
- Determine the threshold for appropriate linkage quality
- Select pairs with a weight above the threshold



Probabilistic linkage

- m_i is the probability that variable i has the same value given that both records are from the same unit
- u_i is the probability that variable i have the same value given that both records are not from the same unit

 m_i and u_i are estimated by means of an EM-algorithm and are therefore approximations



Probabilistic linkage

Weights are determined by:

$$w_{i} = \begin{cases} \ln\left(\frac{m_{i}}{u_{i}}\right) & \text{if value i is similar} \\ \ln\left(\frac{1-m_{i}}{1-u_{i}}\right) & \text{if value i is different} \end{cases}$$

$$w = \sum_{i} w_{i}$$

Choose threshold in such a way that the number of false negatives and false positives are minimised



The MDF-correction

$$\tilde{N}_{\text{our}} = \frac{N_{\text{tot}}}{\hat{\varepsilon}_{\text{tot}} - \hat{\varepsilon}_{\text{tot}} - (\alpha \, \hat{\varepsilon}_{\text{tot}} \hat{\varepsilon}_{\text{tot}} - \beta \, (\hat{\varepsilon}_{\text{tot}} - \hat{\varepsilon}_{\text{tot}} - 2\hat{\varepsilon}_{\text{tot}} \hat{\varepsilon}_{\text{tot}}))} \,, \, \, \text{where}$$

$$\hat{\tau}_{\text{LMDF}} = \frac{2\beta x_{\text{L}}^* + \beta x_{\text{L}}^* + \beta x_{\text{L}}^* + \beta x_{\text{L}}^* - x_{\text{L}}^*}{(2\beta - \alpha)(x_{\text{L}}^* + x_{\text{L}}^*)} \quad \text{and} \qquad \hat{\tau}_{\text{LMDF}} = \frac{2\beta x_{\text{L}}^* + \beta x_{\text{L}}^* + \beta x_{\text{L}}^* + \beta x_{\text{L}}^* - x_{\text{L}}^*}{(2\beta - \alpha)(x_{\text{L}}^* + x_{\text{L}}^*)}$$

		first	
		yes	no
second	yes	n _{1,1}	n _{0,1}
	no	n _{1,0}	n _{o,o}

 α = probability that a true match is linked

 β = probability that a false match is accidentally linked

The MDF-correction

Three problems to solve:

- MDF is developed for only two sources
- MDF is available for models without covariates
- We used several blockings for the probabilistic linkage The estimation of the α and β is not straightforward
- Ignoring this problem leads to implausible results: in all scenarios for removing erroneous captures the number of missed usual residents are negative



Conclusions

- CRC assumes among else perfect linked sources
- Two ways to correct: different scenarios and MDFcorrection
- Theoretically the MDF-correction is superior
- In practice several problems have to be solved
- We will work on that the coming months

