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Carbonell price index and Carbonell quantity index

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Abstract

This paper demonstrates that Paasche price and quantity indices show a deviation factor in the same fashion as Laspeyres price and quantity indices, and the comparison factors between Paasche and Laspeyres are complementary.

Carbonell Price Index and Carbonell Quantity Index are also proposed and concluded by showing the limits of composite price indices.

*Keywords:*Price Index, Quantity Index, Paasche Price Index, Paasche Quantity Index, Laspeyres Price Index, Laspeyres Quantity Index, Carbonell Price Index, Carbonell Quantity Index.

AMS Classification: 62 Statistics.

Índice de Precios de Carbonell e Índice de Cantidad de Carbonell

Resumen

En este artículo se muestra que los índices de precios y de cantidad de Paasche muestran un factor de desviación de la misma manera que los índices de precios y cantidad de Laspeyres, y que los factores de comparación de Paasche con los de Laspeyres son complementarios.

Asímismo, se presenta el Índice de Precios de Carbonell y el Índice de Cantidad de Carbonell y se concluye mostrando los límites de los índices complejos de precios.

Palabras clave: Índice de Precios, Índice de Cantidad, Índice de Precios de Paasche, Índices de Cantidad de Paasche, Índice de Precios de Laspeyres, Índice de Cantidad de Laspeyres, Índice de Precios de Carbonell e Índice de Cantidad de Carbonell.

Clasificación AMS: 62 Estadística.

1. Price index and quantity index

If inflation is a measure of rise in prices of some commodities (484 in Spain) calculated based on CPI variation over a period of time, deflator is similar to this inflation and only in contrast to this, deflator is the measure of rise in prices of all goods and services in the market included in the GDP (Gross Domestic Product, that can be defined as the Added Value of the country production in a period of time.)

Deflator is derived from:

Deflator = Nominal GDP / Real GDP

Apart from this method of calculating deflator there are several price indices that also calculate it. They are Laspeyres and Paasche price indices. Both are weighted indices. Specifically Paasche index is the one that is applied to calculate deflator. We may note the formulas of both the indices:

Laspeyres Price Index:

$$(\sum (Pit * Qio)) / (\sum (Pio * Qio)).$$

Paasche Price Index:

$$(\sum (Pit * Qit)) / (\sum (Pio * Qit))$$

Where o is an initial period and t is a period after initial (i.e., current) time. Sub-index i is commodity type.

The first one measures the relationship between the cost of basket of base year with that of current prices in comparison with that needed to purchase identical basket with base year prices.

While the second one, Paasche index, measures the relationship between the cost of basket of current year with that of current prices in comparison with that needed to purchase identical basket with base year prices. Therefore, each of the indices takes into account a fixed basket but at different times. Laspeyres takes into account the base year and Paasche takes into account the current year. If we do an in-depth analysis of these indices we deduce that for Paasche (Pp) price index:

$$Pp = (\sum (Pit *Qit)) / (\sum (Pio * Qit))$$

$$Pp = 1+ [[(\sum (Pit * Qit)) – (\sum (Pio * Qit))] / (\sum (Pio * Qit))]]$$

$$Pp = 1+ (\sum [(((Pit * Qit) – (Pio * Qit)) / (Pio * Qit)) * ((Pio * Qit) / (\sum (Pio * Qit)))])$$

Using the same above-mentioned steps in the case of Laspeyres (Pl) price index, we get:

$$Pl = 1+ (\sum [(((Pit * Qio) – (Pio * Qio)) / (Pio * Qio)) * ((Pio * Qio) / \sum (Pio * Qio))])$$

We remind Paasche and Laspeyres Quantity Index:

Paasche Quantity Index:

$$[\sum (Pit * Qit)] / [\sum (Pit *Qio)]$$

Laspeyres Quantity Index:

$$[\sum (Pio * Qit)] / [\sum (Pio * Qio)]$$

On the other hand, we shall make a new contribution on Paasche and Laspeyres indices which I present as a fact.

If Nominal GDP = Base year GDP + GDP variation due to prices

+ GDP variation due to quantities

In principle, there are two ways of doing it: by resorting to comparison factors of Paasche price and quantity index or through comparison factors of Laspeyres price and quantity index as follows. However so that we obtain the equality it must exist a factor of deviation and we have to explain the sign, what we do in another point of the paper.

· Using comparison factors of Paasche index:

Nominal GDP = Base year GDP + [∑ (Pit * Qit) - ∑ (Pio * Qit)]

+ [∑ (Pit * Qit) - ∑ (Pit * Qio)]

- Deviation Factor

· Using comparison factors of Laspeyres index:

Nominal GDP = Base year GDP + [∑ (Pit * Qio) - ∑ (Pio * Qio)]

+ [∑ (Pio * Qit) - ∑ (Pio * Qio)]

+ Deviation Factor

This factor what I call as deviation factor is expressed as:

Deviation factor = ∑ [(Pit – Pio) * (Qit – Qio)]

This factor can be understood as commodities whose quantity and prices have changed (increased or decreased) at the same time.

Deviation factor proof:

Comparison factors of Laspeyres + deviation factor

$$= \sum (Pit * Qio) - \sum (Pio * Qio) + \sum (Pio * Qit) - \sum (Pio * Qio) + \sum [(Pit - Pio) * (Qit - Qio)]$$

$$= \sum (Pit * Qio) - \sum (Pio * Qio) + \sum (Pio * Qit) - \sum (Pio * Qio) + \sum (Pit * Qit - Pit * Qio - Pio * Qit + Pio * Qio)$$

$$= \sum (Pit * Qio) - \sum (Pio * Qio) + \sum (Pio * Qit) - \sum (Pio * Qio) + \sum (Pit * Qit) - \sum (Pit * Qio) - \sum (Pio * Qit) + \sum (Pio * Qio)$$

$$= \sum (Pit * Qit) - \sum (Pio * Qio)$$

The resulting index is:

$$\sum (Pit * Qit) / \sum (Pio * Qio)$$

Or put in other words based on what we have just seen:

Nominal GDP = Base GDP + Comparison factors of Laspeyres + deviation factor

Which implies:

$$\sum (Pit * Qit) = \sum (Pio * Qio) + \sum (Pit * Qit) - \sum (Pio * Qio)$$

$$\sum (Pit * Qit) = \sum (Pit * Qit)$$

However, for the calculation of nominal GDP there are two solutions because these are basically two complementary indices combining alternatively comparison factors of Paasche and Laspeyres prices and quantities:

Nominal GDP = Base GDP + Paasche price variation + Laspeyres quantity variation

Which implies:

Nominal GDP = Base GDP + (Nominal GDP – Real GDP) + (Real GDP – Base GDP)

$$\sum (Pit * Qit) = \sum (Pio * Qio) + [\sum (Pit * Qit) - \sum (Pio * Qit)] + [\sum (Pio * Qit) - \sum (Pio * Qio)]$$

Hence:

Alternatively, we shall refer the expression $\sum (Pit * Qio)$ as Parareal GDP because it is a factor reverse to Real GDP.

Synthetically,

Nominal GDP = $\sum (Pit * Qit)$

Base GDP = $\sum (Pio * Qio)$

Real GDP = $\sum (Pio * Qit)$

Parareal GDP = $\sum (Pit * Qio)$

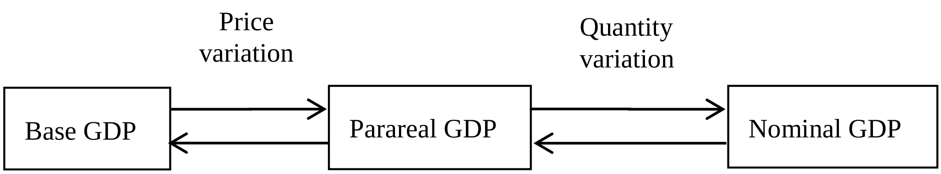
The second method is as per below:

Nominal GDP = Base GDP + Laspeyres price variation
+ Paasche Quantity variation

Which implies:

Nominal GDP = Base GDP + (Parareal GDP – Base GDP) + (Nominal GDP – Parareal GDP)

$$\sum (Pit * Qit) = \sum (Pio * Qio) + [\sum (Pit * Qio) - \sum (Pio * Qio)] + [\sum (Pit * Qit) - \sum (Pit * Qio)]$$



There are many ways to calculate deviation factor. I state the first one with respect to comparison factors of Laspeyres:

$\sum (Pit * Qit)$ – comparison factors of Laspeyres index should be equal to

$$\sum (Pio * Qio).$$

We may note:

$$\sum (Pit * Qit) - [[\sum (Pit * Qio) - \sum (Pio * Qio)] + [\sum (Pio * Qit) - \sum (Pio * Qio)]]$$

$$= \sum (Pit * Qit) - \sum (Pit * Qio) + \sum (Pio * Qio) - \sum (Pio * Qit) + \sum (Pio * Qio)$$

$$= \sum (Pit * Qit) + [2 *(\sum (Pio * Qio))] - \sum (Pit * Qio) - \sum (Pio * Qit)$$

$$= \sum \mathbf{(Pio * Qio)} + \sum (Pit * Qit) + \sum (Pio * Qio) - \sum (Pit * Qio) - \sum (Pio * Qit)$$

Therefore, deviation factor is:

$$\sum (Pit * Qit) + \sum (Pio * Qio) - \sum (Pit * Qio) - \sum (Pio * Qit)$$

Which means:

$\sum (Pit * Qit) = \sum (Pio * Qio)$ + Comparison factors of Laspeyres

+ Deviation factor

Deviation factor by itself could be either positive or negative depending on data. However, this procedure has allowed us to see what to do, either to add or subtract the deviation factor no matter if it is positive or negative.

Let's now deal with respect to comparison factors of Paasche:

∑ (Pio * Qio) + comparison factors of Paasche should be equal to: ∑ (Pit * Qit). We may note:

$$\sum (Pio * Qio) + [\sum (Pit * Qit) - \sum (Pio * Qit)] + [\sum (Pit * Qit) - \sum (Pit * Qio)]$$

$$= \sum (Pio * Qio) + 2 * (\sum (Pit * Qit)) - \sum (Pio * Qit) - \sum (Pit * Qio)$$

$$= \sum \mathbf{(Pit * Qit)} + \sum (Pio * Qio) + \sum (Pit * Qit) - \sum (Pio * Qit) - \sum (Pit * Qio)$$

Which means:

$$\sum (Pit * Qit) = \sum (Pio * Qio) + \text{Comparison factors of Paasche} - \text{Deviation factor}$$

Therefore, as we have seen in the above cases if we start with Base GDP and use comparison factors of Laspeyres to obtain Nominal GDP, deviation factor must be added no matter if it is positive or negative whereas if we use comparison factors of Paasche, deviation factor must be subtracted.

Equivalent expression forms of deviation factor

Below are equivalent expressions of the same deviation factor:

- ∑ [(Pit – Pio) * (Qit – Qio)]
- ∑ (Pit * Qit) - ∑ (Pit * Qio) - ∑ (Pio * Qit) + ∑ (Pio * Qio)
- [∑((Pit – Pio) * Qit)] – [∑ ((Pit – Pio) * Qio)]
- [∑ ((Qit – Qio) * Pit)] – [∑ ((Qit – Qio) * Pio)]

2. Carbonell price index

An assessment has to be made. Having identical quantities between base period and current period for each commodity type what we obtain is as follows:

Base GDP + Price variation = Nominal GDP

That is to say, if we take into account comparison factors of Paasche we will get:

$$\sum (Pio * Qio) + \sum (Pit * Qit) - \sum (Pio * Qit) = \sum (Pit * Qit)$$

Instead if we take into account comparison factors of Laspeyres we will get:

$$\sum (Pio * Qio) + \sum (Pit * Qio) - \sum (Pio * Qio) = \sum (Pit * Qit)$$

In both cases equality is fulfilled because Qit = Qio, therefore ∑ (Pio * Qio) = ∑ (Pio * Qit) and ∑ (Pit * Qit) = ∑ (Pit * Qio).

Given that base period and current period quantities are not the same in general, monetary quantity comes into place whose purpose is to complement price variation as we have seen previously. So we proceed with the following:

Base GDP + Variation = Nominal GDP

Base GDP + Price variation + monetary quantity variation = Nominal GDP

Therefore, necessarily

(Base GDP + variation) / Base GDP = Nominal GDP / Base GDP

= (Base GDP + price variation + monetary quantity variation)/ Base GDP = Nominal GDP / Base GDP

= (Base GDP / Base GDP) + (price variation / Base GDP) + (monetary quantity variation/ Base GDP) = Nominal GDP / Base GDP

$$= (\sum (Pio * Qio) / ((\sum (Pio * Qio))) + (((\sum (Pit * Qit) - \sum (Pio * Qit)) / \sum (Pio * Qio)) + (((\sum (Pio * Qit) - \sum (Pio * Qio)) / \sum (Pio * Qio)) = \sum (Pit * Qit) / \sum (Pio * Qio)$$

Where is Carbonell Price Index derived from:

$$\text{Carbonell Price Index} = ((\sum (Pit * Qit) - \sum (Pio * Qit)) / \sum (Pio * Qio))$$

I call it an index because it is an indicator that is used to compare data although the neutral value is not 1 but 0.

There is a certain paradox as in order to consider the procedure of division by the sum of the nominator that subtracts we necessarily arrive at an index which does not consider it as an index. But if we choose to consider this as the Paasche index does we do not consider $(\sum (Pit * Qit) / \sum (Pio * Qio))$.

That is to say if $(\sum (Pit * Qit) / (\sum (Pio * Qit))$ is complied then $(\sum (Pit * Qit) / \sum (Pio * Qio))$ is not satisfied and if $(\sum (Pit * Qit) / \sum (Pio * Qio))$ is complied then $(\sum (Pit * Qit) / (\sum(Pio * Qit))$ is not satisfied hence undecidability is generated whether to choose Paasche Price Index or Carbonell Price Index.

The disadvantage of Paasche Price Index is that it fabricates the base with which it compares when Qio is transformed to Qit, then, what is the point of comparing the prices if we are fabricating the base? On the other hand, Carbonell price index does not do so as it respects the base even though a reinterpretation of inflation values is required.

However, the paradox and the undecidability vanishes when instead of having the index based on neutral value as 0 if we base it on neutral value as 1 then

$$\text{Carbonell price index} = (\sum (Pio * Qio) + \sum (Pit * Qit) - \sum (Pio * Qit)) / \sum (Pio * Qio)$$

In this case the afore-mentioned paradox loses its sense as the numerator is more complex, simply there is no second sum which is subtracting. Therefore, one of the conclusions is that if (Base GDP + variation) / Base GDP then we can get an index where the denominator is different from a sum that subtracts.

3. Carbonell Quantity Index:

Similar to Carbonell price index we get the Carbonell quantity index based on the other pair of complementary comparison factors.

So we proceed with the following:

(Base GDP + price variation + monetary quantity variation)/ Base GDP = Nominal GDP / Base GDP

= (Base GDP / Base GDP) + (price variation / Base GDP) + (quantity variation/ Base GDP) = Nominal GDP / Base GDP

= (∑ (Pio * Qio) / (∑ (Pio * Qio)) + ((∑ (Pit * Qio) - ∑ (Pio * Qio)) / ∑ (Pio * Qio))

+ ((∑ (Pit * Qit) - ∑ (Pit * Qio)) / ∑ (Pio * Qio))

= (∑ (Pit * Qit) / ∑ (Pio * Qio))

Carbonell Quantity Index is derived from:

Carbonell Quantity Index = ((∑ (Pit * Qit) - ∑ (Pit * Qio)) / ∑ (Pio * Qio))

As in the case of Carbonell price index we can also express Carbonell quantity index with neutral value equal to 1 as follows:

Carbonell Quantity Index = ((∑ (Pio * Qio) + ∑ (Pit * Qit) - ∑ (Pit * Qio)) / ∑ (Pio * Qio))

4. Conclusion

Paasche price index serves to compare what we have spent in a given year at current prices with what we have spent in the same year at constant prices. However illusory inflation is calculated because the sense of inflation when quantity is taken into account is to compare one by one all the prices. However this index does not compare the prices one by one whereas it takes the same current amount. Therefore it is changing the base as comparing prices one by one loses its sense. The same is true with Laspeyres Price Index which also changes the quantities by having them equated in this case with the base quantity.

In this sense we can say that Carbonell Price Index does not vary the base quantity.

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