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Invited Article

# Speech at the National Statistics. Award 2022 Ceremony

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**Abstract:** This work presents the speech given at the 2022 National Statistics Award ceremony, summarizing the main scientific contributions of the recipient, Professor Enrique Castillo. These contributions include significant advancements in the field of extreme value statistics, where he provides analytical and graphical methods for identifying tail types, conditional specification, Bayesian networks, addressing compatibility issues, sensitivity analyses in optimization problems with closed-form solutions, solving linear systems of inequalities, demonstrating that polytopes are the unique bounded solutions, fatigue models based on properties, especially the S-N and crack growth models, which provide the only models satisfying certain necessary compatibility conditions. Additionally, it covers probabilistic safety analyses of nuclear power plants, roads, and railways, allowing the assessment of risks using statistical models that include thousands of variables, as well as applications in artificial intelligence and Bayesian methods that expand the range of possible solutions by considering mixtures of much more limited distribution families.

**Keywords:** Extreme values, conditional distribution specification, Bayesian networks, operations research, fatigue models, nuclear safety, road and railway safety, artificial intelligence, Bayesian methods.

**MSC:** 60E05, 62P99

## 1 Introduction

By the express invitation of José María Sarabia Alegría, editor of the Spanish Journal of Statistics at the National Institute of Statistics (INE), I am writing this article in which I present the speech given upon receiving the National Statistics Award 2022, which was awarded to me. First and foremost, I would like to express my gratitude for the invitation, which I gladly accept.

The ceremony took place at the headquarters of the Royal Academy of Sciences in Madrid and was presided over by the Honorable Secretary of State, the Honorable President of the Royal

 

Academy of Exact, Physical, and Natural Sciences, and the Illustrious Lady President of the National Institute of Statistics. It was attended by the Directors-General of the Institute, the Honorable President of the Royal Academy of Engineering, the Director of CEDEX, several academic colleagues, members of the Jury, the Lady President of the Spanish Society of Statistics and Operations Research (SEIO), as well as invited guests, family, and friends.

Videos of the ceremony can be seen in (https://www.youtube.com/watch?v=FXriLHlj0_I) or in (https://youtu.be/QbqaHYp4Sz0).

I will now proceed to reproduce my speech.

## 2 Previous acknowledgments

First and foremost, I want to express my gratitude to the National Institute of Statistics (INE) for the significant social work it carries out and for establishing and awarding the National Statistics Awards. The existence of these awards reflect a special sensitivity towards both theoretical and applied science. I also want to thank them for selecting me this year for this distinction, considering that there must have been other candidates who would have also been deserving recipients of this award.

In this speech, I aim to highlight some of the key contributions to the field of statistics in which I have had the privilege to participate, alongside my group of colleagues, collaborators, and students.

## 3 Contributions to undergraduate and doctoral programs and received grants and scholarships.

I completed my first university degree in Civil Engineering in June 1969, just as the Doctorate programs were being introduced for the first time in Engineering Schools.

Until then, research in these schools was conducted in prestigious institutions such as CEDEX, see figure 1, or the Eduardo Torroja Institute, and students were awarded a Doctorate degree after 7 years of study and a complex final project, akin to a doctoral thesis. It was during this time that Doctorate programs were initiated in Engineering Schools, aligning them with those in Science faculties. I dedicated significant effort to their launch and implementation

At that moment, we were fortunate to have the support of great engineers and researchers, among whom I want to highlight Professor Jiménez Salas, see 2, who was at CEDEX and was a Full Member of the Royal Academy of Sciences (RAC). In that first year, several of us enrolled in the Doctorate program.

One day, this professor received a visit from Professor Raymond J. Krizek, see figure 2, from Northwestern University, who was looking for students with a strong mathematical background. The result was that I, who had never considered going abroad for a doctorate, found myself with a free scholarship to that university and an exceptional thesis advisor. So, three months later, after getting married, I left for the United States.

Needless to say, that event changed my life and gave me a new perspective on the world. After completing my Doctorate there, I returned to Spain, and during the 72-73 academic year, I completed the coursework and thesis for the Spanish Doctorate, which culminated with an Outstanding Award. Professor Jiménez Salas offered me the responsibility of collecting the necessary soil samples for the foundation of the Almaraz Nuclear Power Plant.

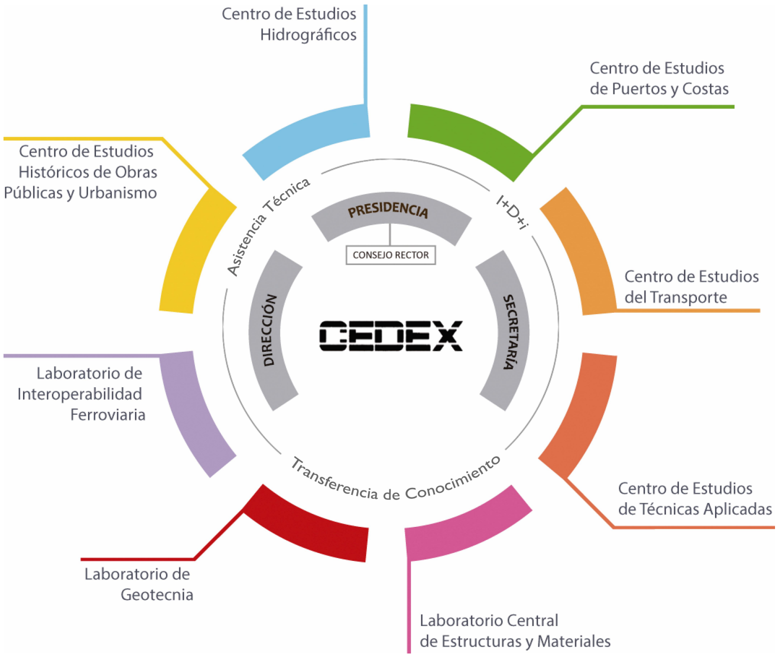


Figure 1: The organizational chart of the Center for Studies and Experimentation of Public Works (CEDEX)

Figure 2: Photos of J. A. Jiménez Salas at the School of Civil Engineering in Madrid and of Raymond J. Krizek's entry in the Hall of Fame at the University of Maryland.

### 3.1 Contributions to the School of Civil Engineering, Waterways, and Ports in Santander

At that time, four full-time professor positions became available at the School of Civil Engineering in Santander, see figure 3, and three of us from the same graduating class, along with one colleague from Santander, applied and were appointed.

Soon after, we had to take on significant roles at the sub-director and director levels, which we distributed without any friction. We were also tasked with improving the undergraduate courses and, notably, launching the doctoral programs.


Figure 3: The School of Civil Engineering, Waterways, and Ports of Santander"

### 3.2 Contributions to the School of Civil Engineering, Waterways, and Ports in Ciudad Real


Figure 4: Rectorate of the University of Castilla-La Mancha

Later on, I collaborated with Professor José María Ureña in founding the School of Civil Engineering at UCLM (University of Castilla-La Mancha) in Ciudad Real, see figure 4, dedicating two years to its establishment.

As I had already discovered that virtually all problems were stochastic, I paid special attention to the Statistics subject in both schools. I was well aware of its importance and its significant role in addressing the challenges we would encounter.

### 3.3 Doctoral Thesis Supervision

Over these years, I have supervised 42 doctoral theses, all of them related to statistical and optimization problems, distributed as follows:

- 25 Civil Engineers,
- 10 Mathematicians
- 3 Industrial Engineers,
- 2 Computer Scientists
- 1 Medical Doctor, and
- 1 Economist.

Among these, 15 of the doctoral candidates were women, despite the fact that I never had a female colleague in the engineering field during my academic career.

### 3.4 List of publications

Over these years, I have also published more than 500 works in which Statistics and modeling play a very relevant role, distributed as follows:

- 14 books in English (published in the top collections by Springer (6), Wiley (3), Elsevier (2), Academic Press, Marcel Dekker, and Kluwer),
- 16 books in Spanish.
- 275 publications in journals with impact.
- 205 publications in conferences.
- 42 various publications.

### 3.5 My first Statistics book

At the School in Santander, I took on the responsibility of teaching Statistics, and I was fortunate to learn from a book by Professor Sixto Ríos, see figure 5, who was an Academic Member of the RAC Institution at that time, during a period when accessing information from books and journals in Spain was not easy. I owe my enthusiasm for Probability and Statistics to him.


Figure 5: Professor Sixto Ríos at his induction as a Full Member of the RAC and his book "Statistical Methods.

I proudly mention the influence of this book, (Ríos, 1967), on my early Statistics knowledge because it has been one of the 5 books that have shaped my main research directions.

In these nearly 50 years of teaching statistics, I have published several books for education in Spanish, some on my own and others with my students. These books have been used in the education of many students at various universities, a fact that is often brought to my attention and fills me with satisfaction.

### 3.6 The Complutense University,

I have also had the opportunity to convey to the authors of these 5 books that inspired me, and let them know that a part of any possible success should be considered theirs.

Since I was teaching statistics and it is a mathematical science, I felt it was necessary to study a degree in Exact Sciences as well. I pursued this at the Complutense University of Madrid, see figure 6, specializing in statistics. To do so, I had to travel by sleeper train at night from Santander to Madrid around twenty times, taking exams.


Figure 6: the Faculty of Mathematics at the Complutense University of Madrid (UCM).

I must also mention that all the knowledge and advancements I acquired along the way have been captured in books. I wrote them when I felt the need to share what I had learned and when I had gained the maturity to do so.

### 3.7 Extreme value statistics and the publication of my extreme value books

From the very beginning, I had a keen interest in extreme value statistics. We know that the design of civil engineering works always involves extreme values, both very small and very large, especially when combining the extremes of some variables with the small values of others. It's well-known that the statistics of the tails of distributions are different from those of the means. Just as the beautiful Central Limit Theorem shows us that if we sum many random variables, their sum tends towards normality regardless of the individual distributions, the same holds true for extreme values, which follow the Generalized Extreme Value distribution, regardless of the underlying distribution.

Due to my interest in this field, I quickly came across the book on extremes by the Hungarian professor Janos Galambos, (Galambos, 1978), see figure 7. It was challenging for me to understand, but it had a significant impact on my future research. Because of this, I registered for a NATO conference in Vimeiro, Portugal, which he was attending, and I persuaded him to co-write a book with me and invite me for a sabbatical year at Temple University in Philadelphia. Later, he couldn't collaborate on the book due to compatibility issues with his own work, but I was able to work closely with him.

I ended up writing my book on extremes, which I published in English through Academic Press, see figure 8. It's my most cited book on Google Scholar, with more than 1476 citations.

Figure 7: Janos Galambos photo.

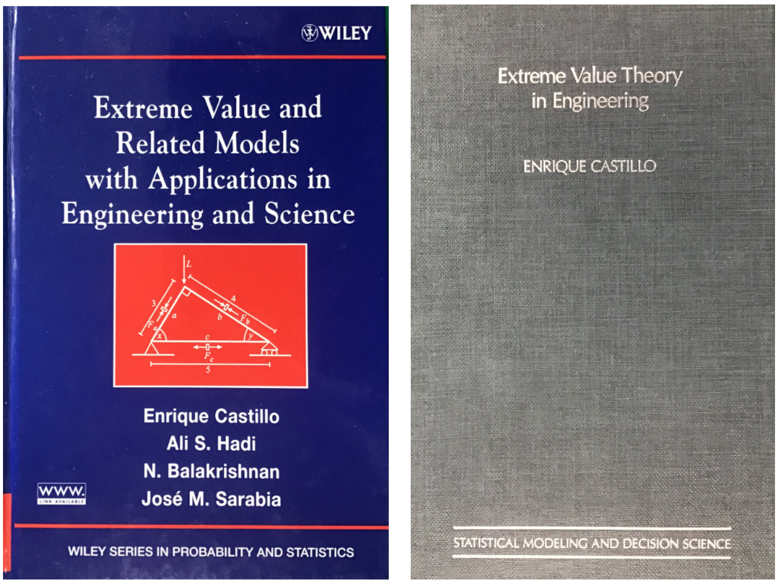


Figure 8: Extreme value books from Academic Press (Series: "Statistical Modeling and Decision Science") and Wiley ("Wiley Series in Probability and Statistics").

I will now present some simple recipes for identifying which distribution to use for fitting extreme value distributions:

To determine whether to use the Weibull, Gumbel, or Fréchet distribution, simply plot the sample on a Gumbel probability paper for maxima or minima, as appropriate, see figure 9, and observe the curvature, as shown in the accompanying figures.

If the curvature is zero, it corresponds to the Gumbel distribution. If it is concave or convex with a vertical asymptote, it will be the Weibull distribution. Otherwise, it will be the Fréchet distribution.

The figure illustrates that when we plot all normalized extreme value distribution quantiles, they cover two regions: one for Weibull and the other for Fréchet, separated by the thick line boundary, which represents Gumbel, see figure 10. It is possible to approximate Gumbel distributions from

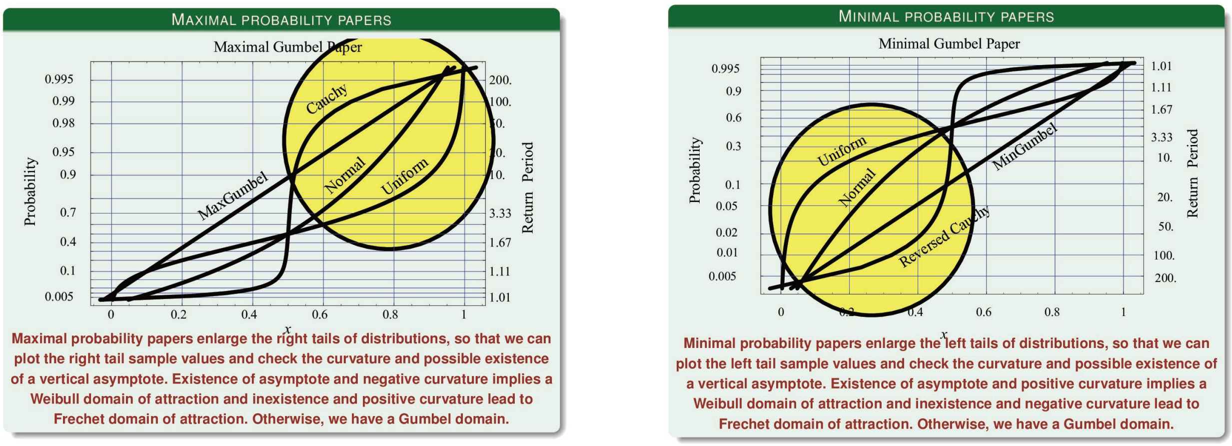


Figure 9: Gumbel probability paper for identifying maximum and minimum distributions.

both families. This simplifies the task of distinguishing between Weibull and Fréchet. However, any

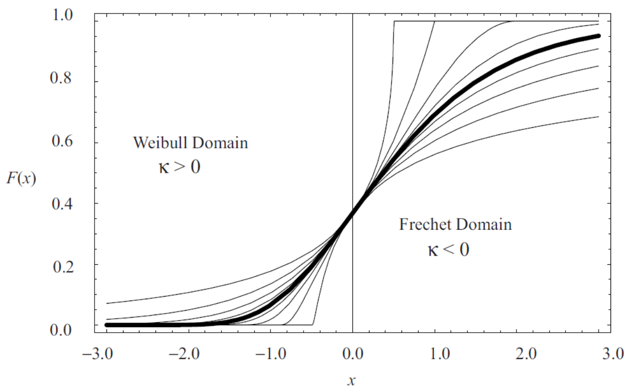


Figure 10: Regions occupied by the different families of extreme value distributions.

bounded extreme value distribution can only be approximated by Weibull, while any unbounded distribution can only be approximated by Fréchet.

Consequently, knowing whether a distribution is bounded or unbounded is sufficient to choose the appropriate family for extreme value modeling. If it is bounded, it belongs to the Weibull family, and if it is unbounded, it belongs to the Fréchet family.

## 3.8 Publications with Janos Galambos

With Janos Galambos, I have co-authored four articles:

1. E. Castillo and J. Galambos. Lifetime regression models based on a functional equation of physical nature. *Journal of Applied Probability*, 24:160–169, 1987.
2. E. Castillo and J. Galambos. Conditional distributions and the bivariate normal distribution. *Metrika, International Journal for Theoretical and Applied Statistics*, 36(3):209–214, 1989.
3. E. Castillo and J. Galambos. Bivariate distributions with Weibull conditionals. *Analysis Mathematica*, 16(1):3–9, 1990.

4. E. Castillo, J. Galambos, and J.M. Sarabia. Caracterización de modelos bivariantes con distribuciones condicionadas tipo gamma. *Estadística Española*, 32(124):439–450, 1990.

and the same number in conferences or similar events.

As a curiosity, I will mention that these publications have allowed José María Sarabia and me, along with Claudi Alsina and a few others, to be among the very few Spaniards with an Erdős number of 2. This is because he co-authored with Erdős, giving him an Erdős number of 1.

### 3.9 Monograph ("Lecture Notes in Statistics") and book from the Springer Series in Statistics.

During my sabbatical with him, one Friday morning, he mentioned that he was going to a conference in a week and asked if I wanted to prepare something to present there. That's when I decided to investigate all bivariate distributions with normal marginals, and I found that, in addition to the bivariate normal distribution, there was another very interesting family. The following week, we worked together and presented the work, which would be the foundation of my research, on conditional distribution specification with Barry Arnold from the University of California and José María Sarabia, see figure 11. It introduced a new methodology for conditional specification that was published in English by Springer in 1999 and has received 476 citations on Google Scholar.

In these works, we demonstrated that conditional distribution specification must be done very carefully, as it can lead to the non-existence of distributions that satisfy all the imposed conditions. Consider this a cautionary note for certain methods that are emerging for conditional specification in Artificial Intelligence environments, which require a deep understanding of this methodology.

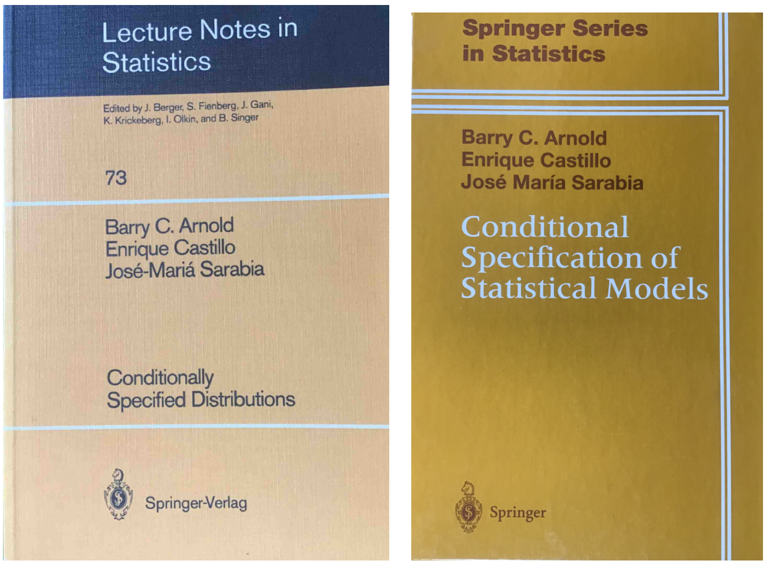


Figure 11: Monograph ("Lecture Notes in Statistics") and conditional specification book ("Springer Series in Statistics").

### 3.10 National Institute of Standards and Technology (NIST)

Moreover, that year, Janos invited me to an expert meeting on extremes at the National Institute of Standards and Technology (NIST), see figure 12. the most important technological center for regulations. During this event, I conducted a course, delivered the keynote address, and participated in the summary roundtable discussion.

Figure 12: National Institute of Standards and Technology (NIST)

### 3.11 Functional equations

By coincidence, I came across the book on functional equations by the Hungarian professor Janos Aczél, (Aczél, 1966), in the library of the ETSICCP in Santander. I was pleasantly surprised by its content and the power of functional equations.

I have many applications of these equations in various statistical, mathematical, engineering, and economic problems. The field of Economics, in particular, has benefited significantly from these equations (see (Castillo and Iglesias, 1997)).

After several years of working with these equations, I published two books in English with my students. One was published by Marcel Dekker in New York in 1992, and the other in the Mathematics in Science and Engineering series by Elsevier in 2005, see figure 13. These books have received 178 and 145 citations on Google Scholar, respectively.

### 3.12 Bayesian networks

In 1988, two significant events took place in the field of Statistics. One was the article by Daniel Spiegelhalter in the Royal Statistical Society, and the other was the publication of Judea Pearl's book, (Pearl, 1988), both on Bayesian networks, see figure 14. I came across both of these works, and they led me to incorporate Bayesian networks into my research areas, which significantly influenced my subsequent years.

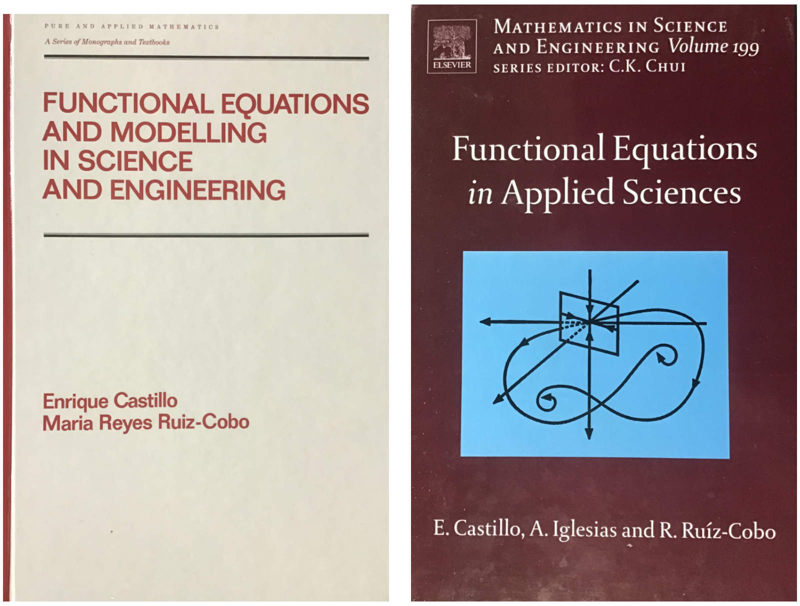


Figure 13: Books on Functional Equations from Marcel Dekker (Series: "Pure and Applied Mathematics") and Elsevier (Series: "Mathematics in Science and Engineering").

Figure 14: Daniel Spiegelhalter and Judea Pearl

As it is well-known, Judea Pearl received the BBVA Foundation Frontiers of Knowledge Award in 2022 for his contributions to Bayesian networks and causality.

### 3.13 Books on Bayesian Networks

After several years of working with these networks in various applications, we published our book on the topic in English with Jose Manuel Gutierrez and Ali S. Hadi in Springer Verlag, New York, in 1997, see figure 15. It has already been cited 1,220 times on Google Scholar.

Our book has received widespread acclaim and has been used in various courses at foreign universities such as the University of California, Duke, Rutgers, Cornell, Iowa State University in the United States, as well as universities in Germany, Finland, Singapore, Brasilia, and at least 7 Spanish universities.

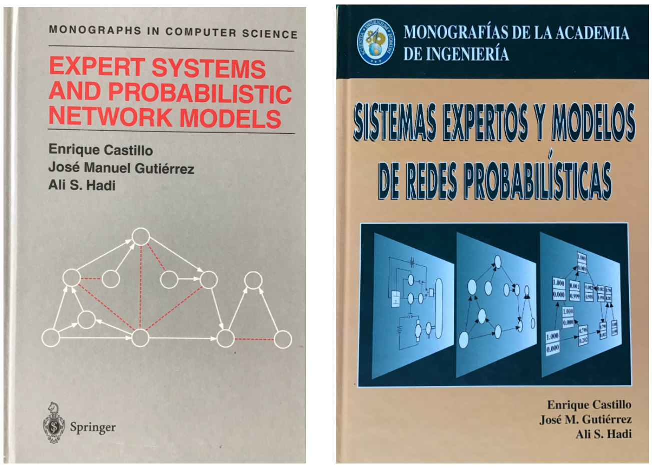


Figure 15: Book on Bayesian Networks in Springer (Series: "Monographs in Computer Science") and its translation (RAI).

### 3.14 Operations Research

As a fifth book, I must mention the book by civil engineer Francisco Jubete on operations research, which contains valuable algebraic material, see (Castillo et al., 1999). This has allowed us to work together in developing revolutionary algebraic methods by considering orthogonality and polarity, see figure 16.

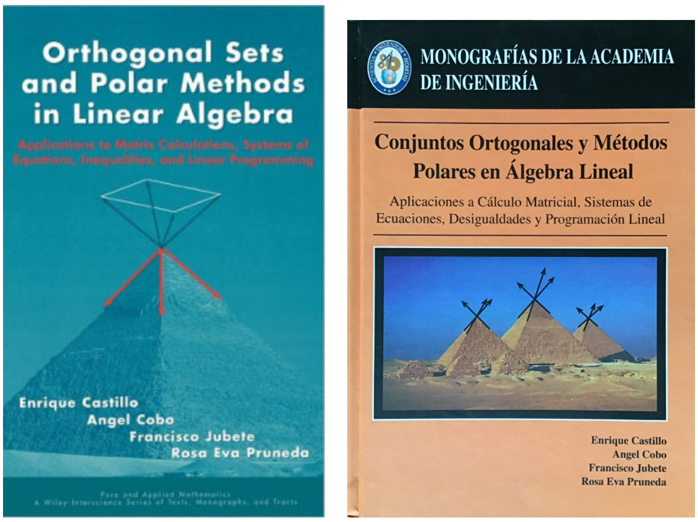


Figure 16: Book on Operations Research in Wiley (Series: "Pure and Applied Mathematics") and its translation (RAI)

An important contribution is in the field of linear systems of inequalities, for which we have provided algorithms to solve them (see (Castillo and Jubete, 2004), (Castillo et al., 1999), (Castillo et al., 1999)).

By being able to solve linear systems of inequalities, which represent polyhedra, i.e., sums of vector spaces, cones, and polytopes (see (Castillo et al., 2001)), and even the simultaneous resolution of subsystems (see (Castillo et al., 2002)), solving linear programming problems becomes trivial. Additionally, it provides all solutions, not just one, as the vast majority of existing methods do.

The optimization problem in all fields of life has led me to carry out significant work in the field of Operations Research. Of particular interest are the results that lead to closed-form formulas for all sensitivities of the objective function and the primal and dual variables with respect to parameters and data.

All of this has been applied to various fields, with a highlight being the optimization of designs and schedules for railway lines, carried out with José María Menéndez and the team from Ciudad Real. This work allowed us to provide the solution of alternating tracks for the Palencia-Santander line, which is at least 30% cheaper than double tracks and with almost the same performance.

The figure 17 shows how, in a 90 km section, the number of daily trains on a single track can be increased from 17 to 34 trains simply by replacing a short section of about 10-12 kilometers with double tracks for crossings, provided that schedules are adjusted and synchronized properly, see figure 17.

It makes no sense to have double tracks for 6 or 8 daily trains, as is the case on several Spanish lines.

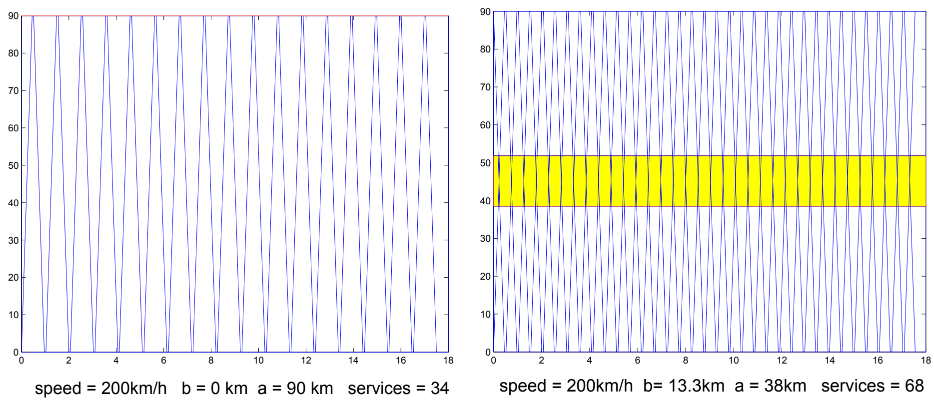


Figure 17: Comparison of the alternating track and double track in terms of their capacities.

## 4 Modeling Material Fatigue using functional equations.

I will now describe some of the contributions of our group to statistical models of material fatigue.

However, I will take this opportunity to describe something much more important, which is our methodology for building models to reproduce real-life problems, observing them, and identifying and replicating their properties.

One day, Professor Alfonso Fernández-Canteli came to my office to consult me about a material fatigue problem, which we solved together. This marked the beginning of a long and fruitful collaboration in Material Strength, especially in S-N curves and crack growth curves.

Functional equations have been key in all of this work, which I came across by chance when I discovered the book by Hungarian Professor Janos Azcél. As I have mentioned, it has been fundamental in my research and that of my group.

I want to emphasize that it is a great error that specific courses on functional equations are not taught in engineering faculties and schools.

As Alfonso had been working at the Polytechnic University of Zurich for 6 years, he introduced me to Professor Bruno Th´urlimann, who funded several short stays, ranging from one week to one month. During these stays, we developed fatigue methods based on properties, meaning that arbitrary assumptions were eliminated or minimized, replaced by fundamental properties. These properties, when expressed as functional equations and then solved, provide the only models that satisfy them. It's a method of great beauty and depth.

Our stay at the Polytechnic University of Zurich in 1985 allowed us to develop, with Bruno Th´urliman and Volker Esslinger, who led the ETH group and the EMPA laboratory, see figure 18, respectively, one of the most important fatigue material research groups in the world. We published our joint model based on properties, followed by a book from Springer and its translation by RAI, see figure 19.

Figure 18: Facilities of the Polytechnic University of Zurich and EMPA in D´ubendorf.

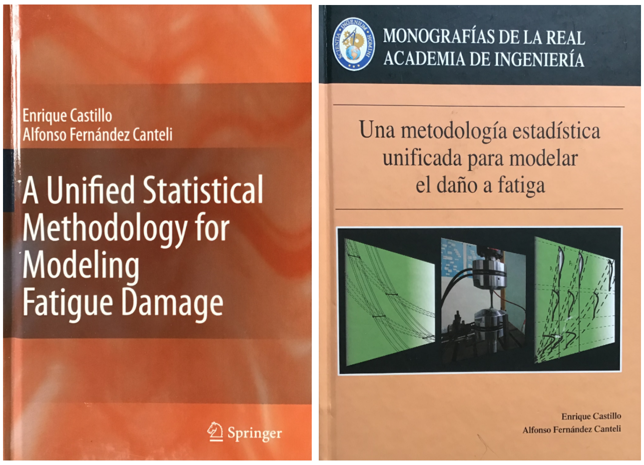


Figure 19: Book on statistical modeling of the fatigue problem (Springer) and its translation (RAI)

During another one of our stays, Alfonso Fernández Canteli and I published a joint article about our fatigue model, with the collaboration of Volker Esslinger and Rolf Kieselbach. Additionally, two of our students joined us during the program and benefited from it. With one of these students, we developed the compatibility model, which became a significant part of her doctoral thesis, which we supervised. Later, she moved to the Fraunhofer Institute in Freiburg, Germany, where she currently works.

Following the tragic and most serious accident involving a high-speed train in Germany (Eschede), the legal proceedings required reports from two laboratories, one independent and one German. Interestingly, these laboratories turned out to be EMPA and the Fraunhofer Institute, where our coauthors Volker Esslinger and Rolf Kieselbach, using our joint method, issued the mentioned report, see figure 20. It was a source of pride and a great surprise for us (see (Castillo et al., 2006)).

The local report was prepared by the Fraunhofer Institute in Freiburg, where our alumna works, and with whom we have collaborated on fatigue-related topics.

Figure 20: Railway accident that occurred in Eschede, Germany)

### 4.1 S-N Models

In materials fatigue, mathematical models are constructed based on experimental data, such as those shown in the figure, to make predictions about the lifespan of materials based on their loadings.

There are two ways to do this:

1. *Data-Driven Approach:* This method involves fitting a model to the data by choosing functions that minimize the differences between predictions and observations. While this approach serves its purpose, especially when interpolating within the data range, providing reasonable predictions if done correctly, it doesn't reveal much about the essence of the problem.
2. *Property-Based Approach:* This approach goes much further, aiming to replicate the mechanisms of fatigue and seeking mathematical expressions or formulas that help us understand and reproduce their behavior, see figure 21. Models generated through this approach often exhibit good quality, extending at times to extrapolations beyond the data range.

These models have been described in detail in (Castillo and Fernández-Canteli, 2009) and have led to software that is freely available to the entire fatigue community. This software allows for the analysis of the described models (see (Fernández-Canteli et al., 2014)).

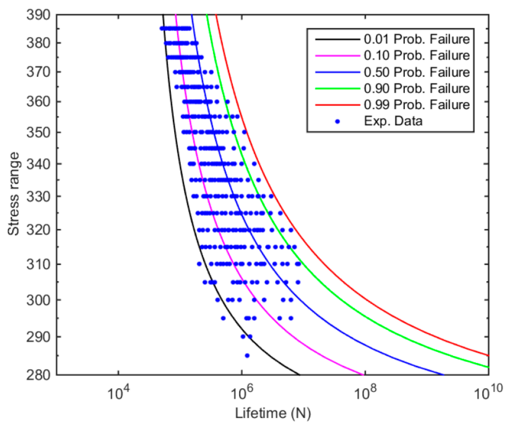


Figure 21: S-N data and adjusted percentile curves.

### 4.2 Fracture growth models. Functional Equations.

As an example of a property-based model construction method, we sought a formula

$$\boxed{\mathbf{a^* = f(a_0, N)}.} \tag{1}$$

that calculates the final crack size, $a^*$ of a specimen with an initial crack size $a_0$ subjected to $N$ loading cycles is as follows:

The property we will use is as follows: It should yield the same result whether we start with $a_0$ and apply $N_1 + N_2$ cycles or if we apply $N_1$ cycles, calculate the associated crack, and, taking this as the initial crack size, apply $N_2$ cycles. In other words, it must satisfy the equation

$$\boxed{\mathbf{f(a_0, N_1 + N_2) = f(f(a_0, N_1), N_2)}.} \tag{2}$$

Which is nothing more than a functional equation in which the unknown is the two arguments function $f$. The general solution of this equation is:

$$\boxed{\mathbf{f(a_0, N) = \boldsymbol{\phi}\left(\boldsymbol{\phi}^{-1}(a_0) + N\right)}} \tag{3}$$

where $\phi$ is an arbitrary invertible function, which is unique except for a translation.

This means that all and only the damage accumulation functions that satisfy this property should be of this form:

$$\boxed{\mathbf{a^* = f(a_0, N) = \boldsymbol{\phi}\left(\boldsymbol{\phi}^{-1}(a_0) + N\right)}.} \tag{4}$$

Enjoy the privilege of now observing and knowing this beautiful result. Also, note that the condition is so strong that it helps us reduce the difficulty of knowing a function of two arguments $f(a_0, N)$ to knowing only a function of one argument $\phi(\mathbf{N})$, because if we know the latter, through (4) we immediately have $f(a_0, N)$.

The figure shows how to interpret the form of the solution:

1. We enter with $a_0$,
2. We calculate $\phi^{-1}(a_0)$,

3. and add $N$,
4. an enter into $\phi$ to obtain the final crack size $a^*$.

if $\phi^{-1}(a_0) = 0$, that is, $\phi(0) = a_0$, Then, from (4), We see that the function $\phi$ directly provides the crack size for $N$ cycles.

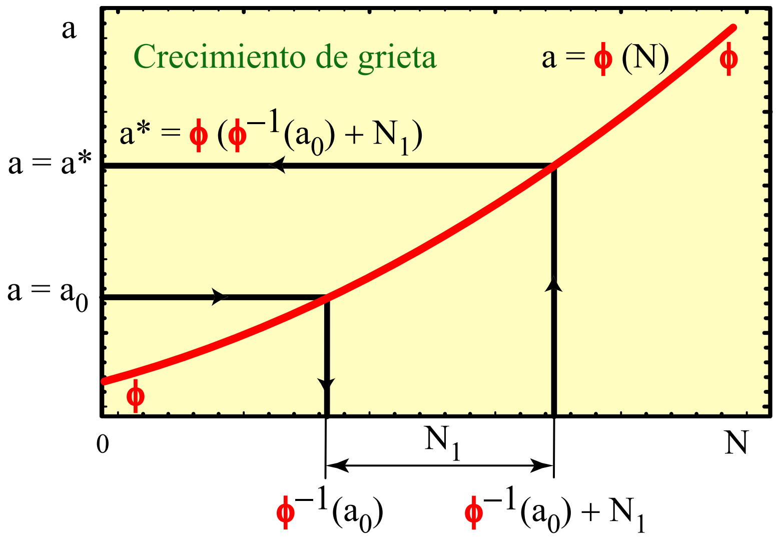


Figure 22: Illustration of the solution to the crack growth problem.

In materials fatigue, the initial aim is to find a model $N = h_1(\Delta\sigma|\sigma_M)$, that gives the lifetime $N$ cycles, for a given stress range $\Delta\sigma = \sigma_M - \sigma_m$ with $\sigma_M$ being constant, as indicated in the upper part of the figure.

As $\Delta\sigma$ increases, $N$, decreases, so each specimen corresponds to a red curve, which shifts to the right as its strength increases. Since it is a destructive test, only one point on the curve is obtained, but it is known that the entire curve exists. The different curves indicate that it is a random problem. To understand the shape of these curves, $N = h_1(\Delta\sigma|\sigma_M)$, we observe a property they must satisfy. Specifically, we examine that if we cut the figure by $\Delta\sigma = cte$ and by $N = cte$, the numbers of curves entering and leaving in the lower-left quadrant must match, which leads to the functional equation, see figure 23:

$$\boxed{\mathbf{F_1}(\boldsymbol{\Delta}\sigma|\mathbf{N}) = \mathbf{F_2}(\mathbf{N}|\boldsymbol{\Delta}\sigma),} \tag{5}$$

where $F_1$ and $F_2$ are the corresponding distribtion functions.

This equation helps describe the fatigue life as a function of stress range and constant mean stress.

If we add the condition of failure by the weakest link and assume that Weibull distributions are appropriate due to their characteristics of being extreme value distributions and bounded, we obtain the functional equation:

$$\boxed{\left(\frac{\mathbf{N} - \mathbf{a_1}(\boldsymbol{\Delta}\sigma)}{\mathbf{b_1}(\boldsymbol{\Delta}\sigma)}\right)^{\mathbf{c_1}(\boldsymbol{\Delta}\sigma)} = \left(\frac{\boldsymbol{\Delta}\sigma - \mathbf{a_2}(\mathbf{N})}{\mathbf{b_2}(\mathbf{N})}\right)^{\mathbf{c_2}(\mathbf{N})}} \tag{6}$$

in six unknown functions, whose general solution is:

$$\boxed{\mathbf{p} = \mathbf{1} - \exp\left[-\left(\frac{(\mathbf{N} - \mathbf{B})(\boldsymbol{\Delta}\sigma - \mathbf{C}) - \lambda}{\delta}\right)^{\beta}\right]} \tag{7}$$

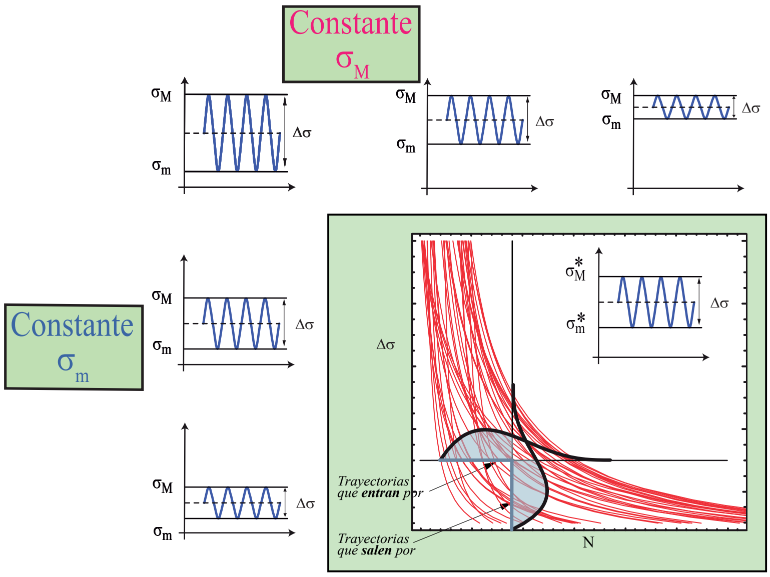


Figure 23: Compatibility of the curves $\Delta\sigma - N$

where $B, C, \lambda, \delta$ and $\beta$ are the parameters and $p$ the percentiles.

All and only the solutions of (7) satisfy the compatibility conditions obtained based on properties.

If instead of fixing $\sigma_M$, $\sigma_m$ is fixed, as on the left side of the figure, the same model is obtained for $N = h_2(\Delta\sigma|\sigma_m)$. Since the cycles in the upper left part are common to both, another compatibility condition arises that must be met.

The figure 24 illustrates this new compatibility condition. The value of $\Delta\sigma$ must coincide in both models for all percentiles $p$, so the intersection points must be on the same horizontal line.

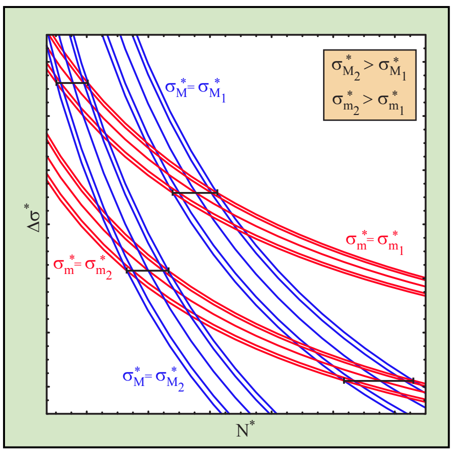


Figure 24: Compatibility of the families of curves for constant $\sigma_M$ and constant $\sigma_m$.

This strong condition leads to the new functional equation:

$$h\left(\frac{(N-B_m(\sigma_m))(\sigma_M-\sigma_m-C_m(\sigma_m))-\lambda_m(\sigma_m)}{\delta_m(\sigma_m)}\right) = h\left(\frac{(N-B_M(\sigma_M))(\sigma_M-\sigma_m-C_M(\sigma_M))-\lambda_M(\sigma_M)}{\delta_M(\sigma_M)}\right)$$

which depends on eight unknown functions, $B_m(\sigma_m)$, $C_m(\sigma_m)$, $\lambda_m(\sigma_m)$, $\delta_m(\sigma_m)$, $B_M(\sigma_M)$, $C_M(\sigma_M)$, $\lambda_M(\sigma_M)$ and $\delta_M(\sigma_M)$ which, when solved, gives us all and only the models that satisfy both conditions.

After solving it, we arrive at the model:

$$\boxed{\mathbf{p}=\mathbf{1}-\exp\left[-(\mathbf{r}(\sigma_\mathbf{m},\sigma_\mathbf{M})+\mathbf{s}(\sigma_\mathbf{m},\sigma_\mathbf{M}))^\beta\right]} \quad (8)$$

where

$$\boxed{\mathbf{r}(\sigma_\mathbf{m},\sigma_\mathbf{M})=\mathbf{C_0}+\mathbf{C_1}\sigma_\mathbf{m}+\mathbf{C_2}\sigma_\mathbf{M}+\mathbf{C_3}\sigma_\mathbf{m}\sigma_\mathbf{M}} \quad (9)$$

$$\boxed{\mathbf{s}(\sigma_\mathbf{m},\sigma_\mathbf{M})=\mathbf{C_4}+\mathbf{C_5}\sigma_\mathbf{m}+\mathbf{C_6}\sigma_\mathbf{M}+\mathbf{C_7}\sigma_\mathbf{m}\sigma_\mathbf{M},} \quad (10)$$

where $C_0, C_1, \ldots, C_7$ are the model parameters.

In this way, we have obtained the model that provides $N$ as a function of both minimum stress, $\sigma_m$, and maximum stress, $\sigma_M$, based exclusively on properties, which illustrates the power of functional equations.

Unfortunately, models that only include one of the stresses $\Delta\sigma$, $\sigma_m$ or $\sigma_M$ are still being used, and even worse, they do not incorporate the indicated compatibility conditions.

As another example of compatibility, the figure presents a crack growth model, see figure 25 that exhibits an important property allowing us to deduce the model using functional equations as well.

The stochastic process model provides the crack size given the number of cycles for three different loadings that combine different minimum and maximum stresses.

The idea is to achieve the same probability density functions for crack sizes with all loadings. Note that the three horizontal groups of three probability density functions each provide the same densities within their group but different ones in different groups, i.e., changing with the damage level.

Finally, in this example, we show how the $S-N$ models are related to crack growth models, see figure 26. In it, you can see how you can transition from one set of curves to another using these models and the failure curve, shown in blue at the bottom.

## 5 Traffic models

Our publications on traffic problems have been numerous at the University of Cantabria, the University of Castilla-La Mancha, and the Hong Kong University of Science and Technology. In particular, we have contributed to traffic monitoring through the intelligent use of sensors, observing vehicle license plates and identifying their routes, as in (Castillo et al., 1999), (Castillo et al., 2001), (Castillo et al., 2002), (Castillo and Jubete, 2004), (Castillo et al., 2005), (Castillo et al., 2006), (Castillo et al., 2007), (Castillo et al., 2010), (Castillo et al., 2010), (Castillo et al., 2013), (Castillo et al., 2015), and (Xu et al., 2016), recreating the traffic evolution, taking congestion into account, and introducing relevant new concepts.

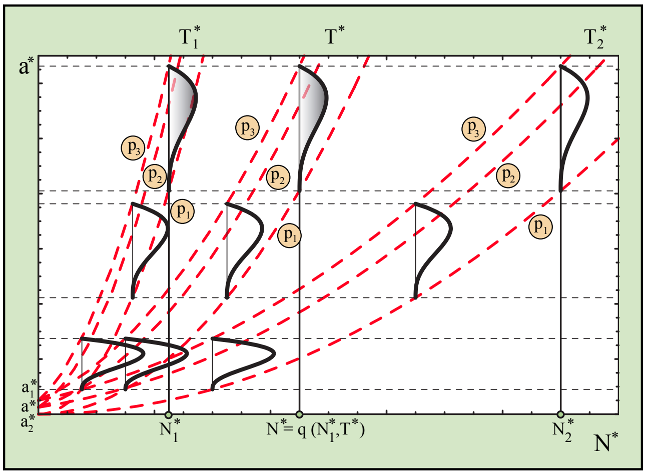


Figure 25: Another compatibility condition for crack growth curves.

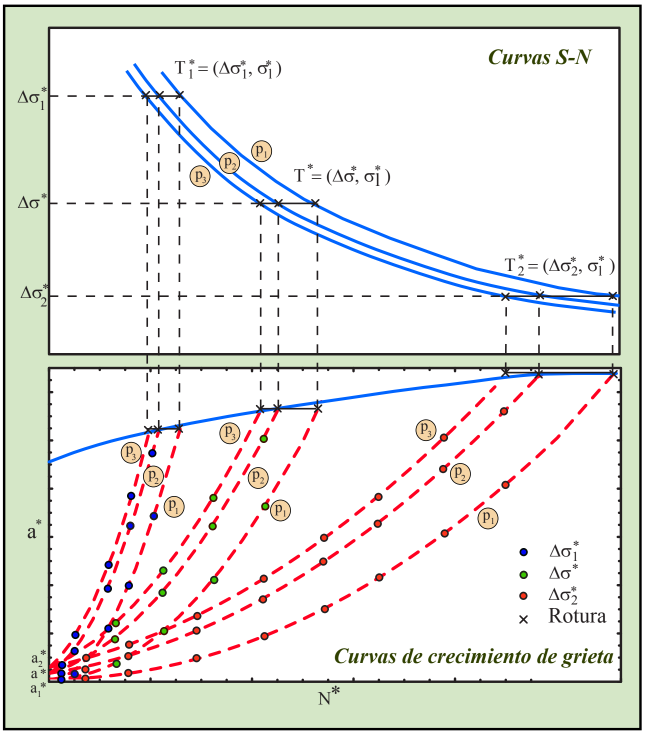


Figure 26: Total compatibility between the S-N models and the crack growth models

## 6 Nuclear Safety. First Probabilistic Safety Analysis in a Spanish Nuclear Power Plant.

As a civil engineer, I have been constantly concerned with the topic of structural safety, having developed several methods to analyze and quantify it. One of the most interesting contributions we have made is demonstrating that methods based on safety coefficients are equivalent to those based on the probability of failure, and that each safety coefficient is associated with its corresponding probability of failure (see (Castillo et al., 2004)).

I was involved in the first probabilistic safety analysis of a Spanish nuclear power plant, Garoña, see figure 27, as co-responsible with Julio González from Nuclenor for this initial study, serving as a statistical expert. Thousands of event sequences that could potentially lead to a plant failure were evaluated, and the probabilities of each of them were quantified, taking actions until these probabilities reached acceptable levels. In particular, human errors were analyzed, and the corresponding corrective measures were designed.

Figure 27: Santa María de Garoña Nuclear Power Plant

# 7 Railway Safety. Alvia Train Accident in Santiago

The fact that I was one of the experts in the trial of the Alvia train accident in Santiago de Compostela in 2013 led me to analyze, for the first time, a probabilistic safety analysis of railway lines at the level of those mandatory in nuclear power plants, replacing the event and fault trees with Bayesian networks, see figure 28, which are far superior to them. I started with Aida Calviño's doctoral thesis and then continued with Zacarías Grande and the group from Ciudad Real.

In addition to applying it to the accident study, we applied it to two Spanish railway lines. This project was the subject of several doctoral theses I supervised, which received national and international awards.

## 7.1 Railway Safety. Eschede Accident

The Santiago accident could have been avoided if these methods had been used.

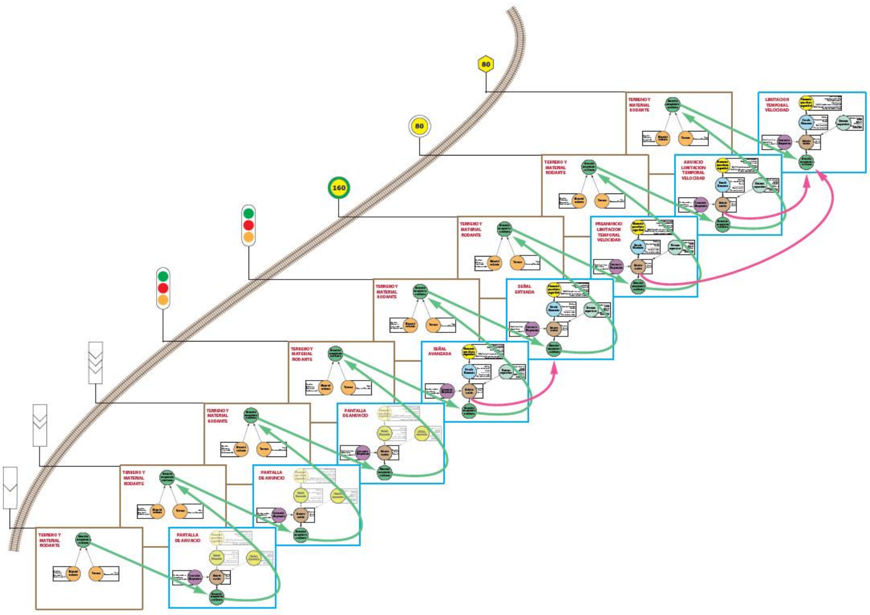


Figure 28: Building the Bayesian Network for a railway line.

It was also the deadliest high-speed rail accident in Germany.

# 8 Road Safety

We also applied this methodology to the study of roads, see figure 29, applying it to three roads in Cantabria and three in Castilla-La Mancha, and it was part of other doctoral theses that I supervised. Four women participated in these awards, and three of them received the national Abertis award for the best theses in the field of transportation and later the international Abertis Award, competing with award-winning individuals from other countries.

## 8.1 Different Types of Items

When traveling along the road, you identify the points and/or sections where decisions need to be made and accidents can occur, which we will refer to as "items.".

LIST OF ITEMS USED IN THE MODEL

- Segments without signs
- Side entrances
- Intersections
- Access lanes
- Roundabouts
- Tunnels
- Viaducts
- Curves
- Slopes

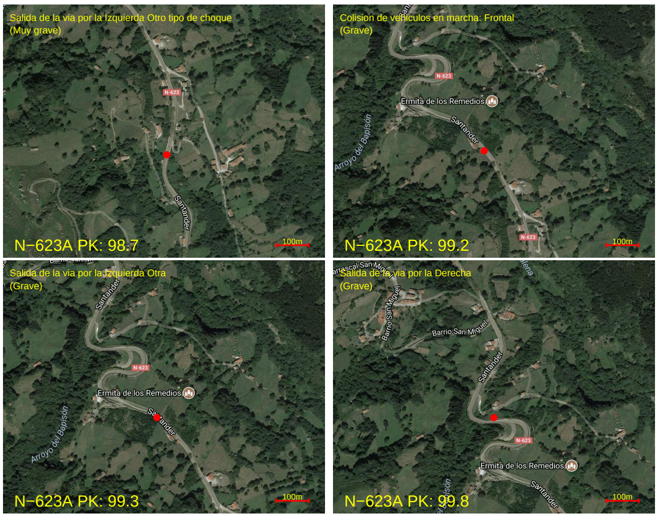


Figure 29: Aerial view of the most dangerous points on the road.

- No overtaking signs
- Speed limit signs
- Traffic lights
- Stop signs
- Give way signs
- Danger/alert signs
- Crosswalks
- Level crossings
- Distracting elements

## 8.2 Used variables

Furthermore, we identify the variables that influence each item, see figure 30.

LIST OF VARIABLES USED IN THE MODEL

1. $D$: Attention
2. $T$: Fatigue
3. $Sd$: Speed decision
4. $Dri$: Driver type
5. $AS$: Decision at a traffic light
6. $Sd$: Decision at a signal
7. $It$: Traffic intensity
8. $Vis$: Visibility
9. $Vt$: Vehicle type
10. $S$: Speed
11. $W$: Weather
12. $SS$: State of a traffic light

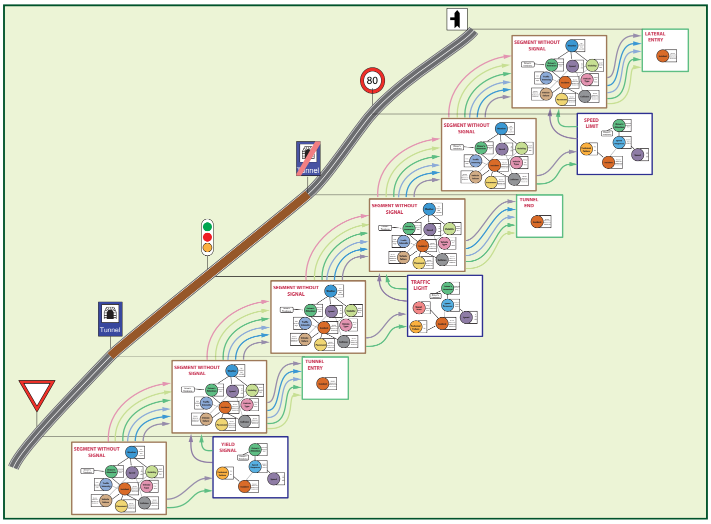


Figure 30: Relationship between the road and its elements with the Bayesian network model.

13. $TF$: Technical failure
14. $V$: Vehicle failure
15. $P$: Pavement failure
16. $Co$: Collision
17. $I$: Incident

## 8.3 Building the Bayesian Network

The process of building the Bayesian network is as follows, see figure 31:

1. Road items are identified.
2. Variables are defined.
3. Variables are assigned to each item.
4. Local dependencies are defined.
5. Conditional distributions are specified.
6. The joint distribution is constructed.

If the variables are independent, the joint distribution is:

$$F(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} F_{1i}(x_i),$$

while if they are dependent, it results in:

$$F(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} F_{2i}(x_i|Pa(X_i)),$$

that is, the product of their marginals or conditionals, respectively.

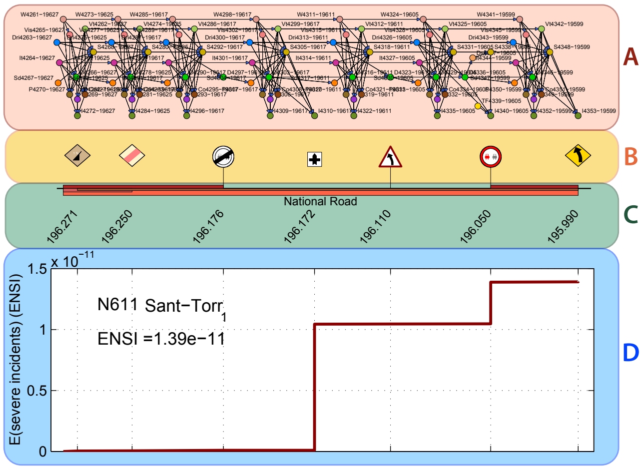


Figure 31: Construction of the Bayesian network

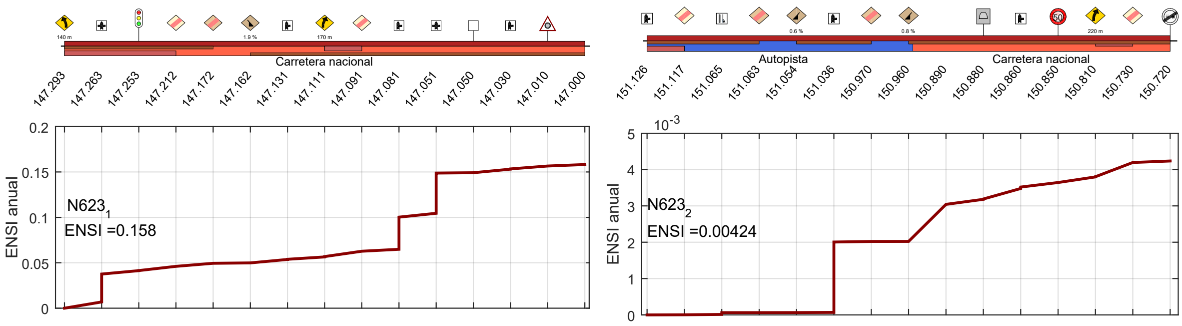


Figure 32: Graphical information provided by the model.

## 8.4 Partitioning of the Bayesian network

The calculation of a road segment with around 7000 variables can take approximately 20 days on a laptop computer.

However, if the network is properly partitioned, see figure 33, the calculation time is greatly reduced (about 8 minutes), which is roughly three orders of magnitude.

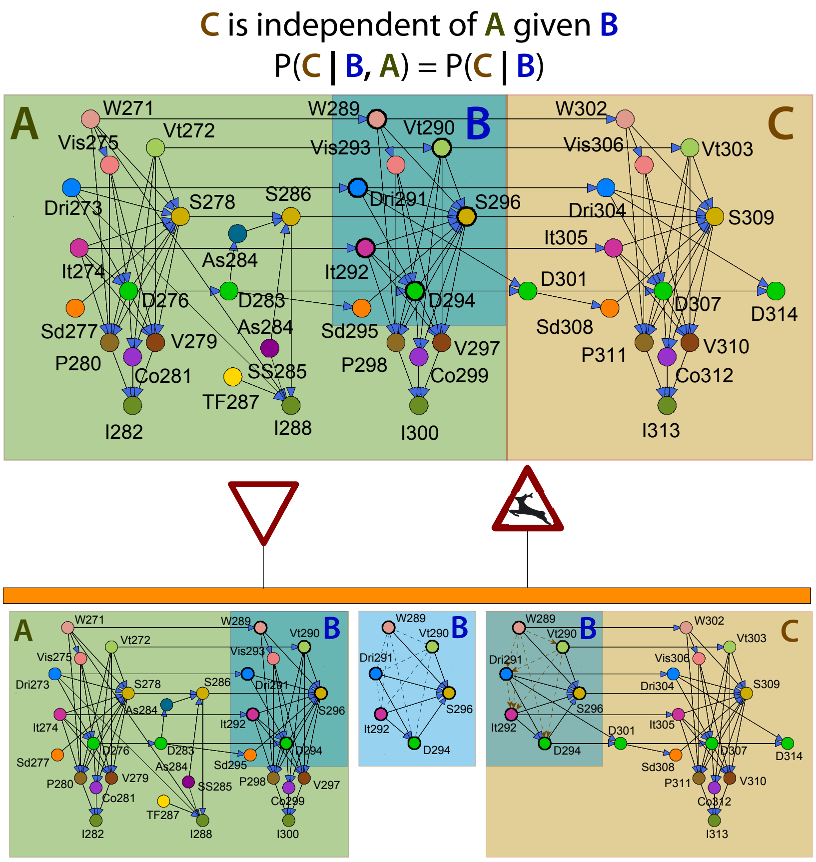


Figure 33: Partitions of the Bayesian network.

Furthermore, the model can send the corresponding Bayesian subnetwork for each segment to the onboard computer to calculate the probabilities for each user at the specific moment of each trip.

Note that conditions vary each day (weather, visibility, traffic intensity, etc.), and so do the risks.

## 8.5 Identification of the most dangerous points on the road.

The program calculates the accident probabilities for all the items and identifies the 20 most dangerous ones, placing them on the map and providing Google Earth images, see figure 34.

Figure 34: Some information about the most risky points, given by the application.

The figure 35 gives(a) An Illustration and location of the 20 most dangerous points along the route, (b) the Location of the most dangerous points, minor, moderate, and severe, using colors and sizes of corresponding circles, and (c) The identification of dangerous points and identification of sections with a high concentration of accidents.

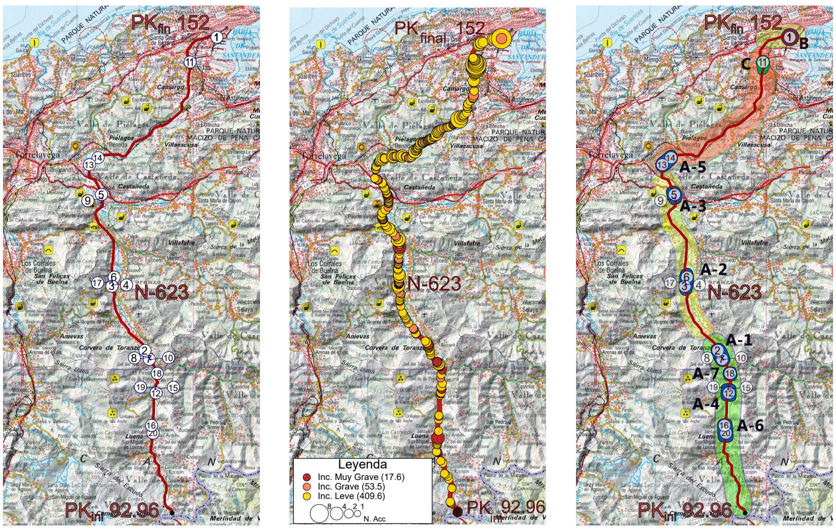


Figure 35: Location of the 20 most dangerous points along the route and identification of sections with a high concentration of accidents.

finaly, figure 36 shows the graphic information illustrated by the application.

This allows marking the items using circles with increasing radii based on their danger level, and also, marking the sections using different colors based on their danger level, see figure 35

## 8.6 Explanation of the causes of accidents

In addition, after an accident has occurred, information about the expected circumstances can be gathered.

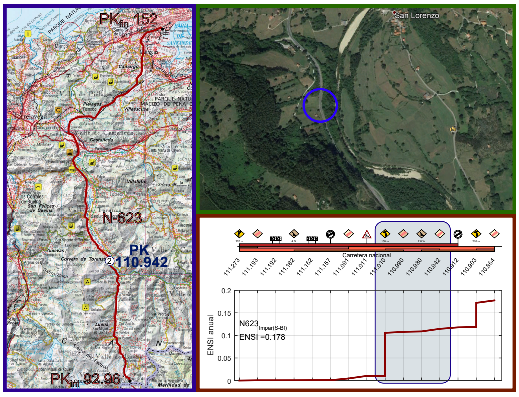


Figure 36: Graphic information illustrated by the application.

In this example of a serious motorcycle accident with cloudy weather, it is observed that such accidents occur at very high speeds, low traffic intensity, and with attentive but inexperienced or reckless drivers, see figure 37.

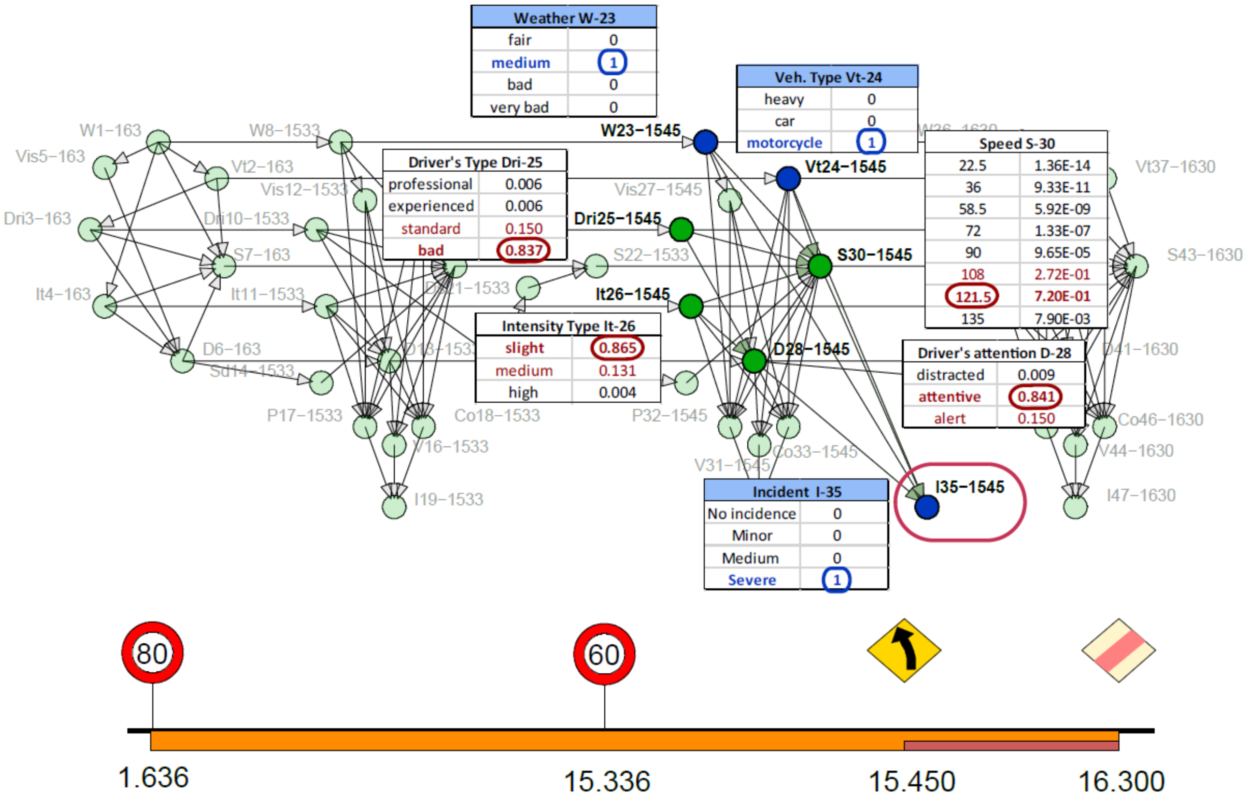


Figure 37: "Bayesian network showing its nodes, links, and conditional probability tables."

## 8.7 Measures to reduce accidents.

After analyzing the hazard, the best improvement actions in terms of results and cost can be suggested, as shown in Table 1.

To do this, the resulting hazard of the improvements is calculated and compared to the current one.

This process can be repeated until the results are satisfactory in terms of cost and safety.

Table 1: Table used in the cost and profitability analysis of safety improvements.

| **Critical Points Analysis** | | | | | |
|---|---|---|---|---|---|
| **Actual Situation** | **N** | **ENSI Annual** | **Annual Incident Probability** | | |
| | | | **Severe** | **Medium** | **Small** |
| **Ascendent (El Escudo - Santander)** | 163 | 1,465 | 1,032 | 2,548 | 8,167 |
| **Descendent (Santander -El Escudo)** | 241 | 2,473 | 1,589 | 4,885 | 27,769 |
| **Total** | 404 | 3,938 | 2,621 | 7,433 | 35,936 |
| **Improved Situation** | **N** | **ENSI Annual** | **Annual Incident Probability** | | |
| | | | **very Severe** | **Severe** | **small** |
| **Ascendent (El Escudo - Santander)** | 110 | 0,706 | 0,406 | 1,819 | 5,502 |
| **Descendent (Santander -El Escudo)** | 157 | 0,884 | 0,383 | 2,590 | 22,146 |
| **Total** | 267 | 1,59052 | 0,789 | 4,409 | 27,649 |
| **Final change** | **N** | **ENSI Annual** | **Annual Incident Probability** | | |
| | | | **severe** | **Medium** | **Small** |
| **Ascendente (El Escudo - Santander)** | 32,5% | 51,8% | 60,7% | 28,6% | 32,6% |
| **Descendent (Santander -El Escudo)** | 34,9% | 64,3% | 75,9% | 47,0% | 20,2% |
| **Total** | 33,9% | 59,6% | 69,9% | 40,7% | 23,1% |

| **TEA and PA Analysis** | | | | | **Required investment (euros 2017)** | | |
|---|---|---|---|---|---|---|---|
| **PA** | **TEA** | **ibitial PK** | **PK End** | **Length** | **Ascendent** | **Descendent** | **Total** |
| **1** | **A-1** | 110,383 | 111,01 | 0,627 | 951,86 | 1444,32 | 2396,18 |
| **2** | **A-2** | 118,196 | 118,97 | 0,774 | 548,784 | 548,784 | 1097,57 |
| **3** | **B** | 146,5 | 147,7 | 1,25 | 16985,112 | | 16985,112 |
| **4** | **A-3** | 127,423 | 127,928 | 0,505 | 973,896 | 1459,008 | 2432,90 |
| **5** | **A-4** | 105,526 | 106,079 | 0,553 | 1137,960 | 466,752 | 1604,71 |
| **6** | **C** | 151,98 | 151,98 | 0,025 | 2158,500 | | 2158,500 |
| **7** | **A-5** | 131.283-131.404 | 132.091-132.212 | 0,242 | 488,784 | 570,816 | 1059,60 |
| **8** | **A-6** | 102.990-103.122 | 101.961-102.000 | 0,171 | 1055,928 | 973,896 | 2029,82 |
| **9** | **A-7** | 108,376 | 109,03 | 0,654 | 488,784 | 488,784 | 977,57 |
| | | | | | **Total Investment** | | **30741,97** |

## 8.8 Return on Investment (ROI) from Improvement Corrections

Next, the benefits of the investment made over 10 years are evaluated, resulting in a 5-year benefit of 21 million euros for an investment of 31,000 euros.

Such high profitability is only possible when there are very inexpensive and efficient improvements. As more improvements are chosen, the cost increases rapidly.

Although we have already developed the software and have used it to analyze the safety of three roads in Cantabria and many others in Castilla-La Mancha and also in Ireland, the Spanish institutions have not known how to take advantage of this tool that is already operational.

| Social Benefit | Accident Reduction | | | | Economic Benefit (Thousand euros 2017) | | | |
|---|---|---|---|---|---|---|---|---|
| Acc. type | Severe | Medium | Small | Total number | Severe | Medium | Small | Total number |
| Collision on the road | 3,45 | 9,05 | 6,02 | 18,52 | 4.830,00 | 1.981,95 | 36,72 | 6.848,67 |
| Curve | 6,97 | 5,21 | 12,46 | 24,63 | 9.758,00 | 1.140,99 | 76,01 | 10.975,00 |
| Collision with animals | 0 | 1,74 | 1,43 | 2,91 | 0,00 | 381,06 | 8,72 | 389,78 |
| Incident on the road | 1,30 | 1,09 | -0,38 | 1,99 | 1.820,00 | 238,71 | -2,32 | 2.056,39 |
| Intersection | 0,00 | 1,01 | 1,39 | 2,40 | 0,00 | 221,19 | 8,48 | 229,67 |
| Lateral Entry | 0,00 | 0,00 | 0,76 | 0,75 | 0,00 | 0,00 | 4,64 | 4,64 |
| Roundabout | 0,00 | 1,66 | 5,94 | 7,60 | 0,00 | 363,54 | 36,23 | 399,77 |
| Pedestrians run over | 0,02 | 0,01 | 0,10 | 0,12 | 28,00 | 2,19 | 0,61 | 30,80 |
| Crosswalk | 0,00 | 0,00 | -0,03 | -0,03 | 0,00 | 0,00 | -0,18 | -0,18 |
| Total | **11,74** | **19,77** | **27,69** | **58,89** | **16.436,00** | **4.329,63** | **168,91** | **20.934,54** |

However, Zacarías Grande, my disciple and co-author of this complex application, see figure 38, has created a company, which together with another Spanish and an Indian company, has just signed a framework agreement with the Indian Minister of Development, through which this new methodology to the roads of India, which with 1.4 billion potential customers, gives us goosebumps. Let's hope that this program is concluded and not just one attempt.

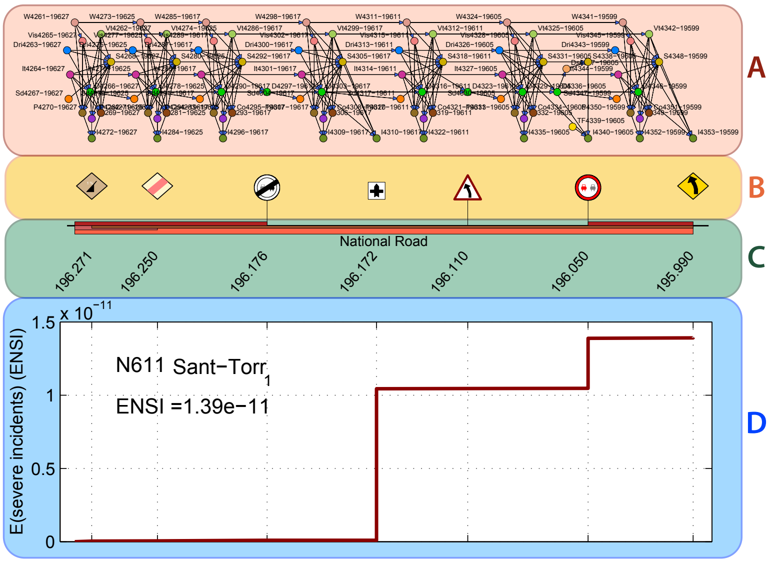


Figure 38: Graphic information provided by the application. Bayesian Network, items present on the road, along with their PKs, and accumulated risk graph, showing their jumps and slopes.

In figure 39 an application is shown that allows you to visually explore the roads, and know the probabilities of accidents when passing through each point on them.

With it you can see:

1. The Bayesian subnetworks associated with the different sections of the road,
2. The corresponding kilometer points.
3. A moving vehicle whose front marks its position on the road.
4. The accident probability diagram of the points passed through, localized or section.
5. A series of buttons that allow you to choose sections using their station, travel time or page.
6. Hiding the video using a button gives you access to the detailed tables of probabilities and risks.

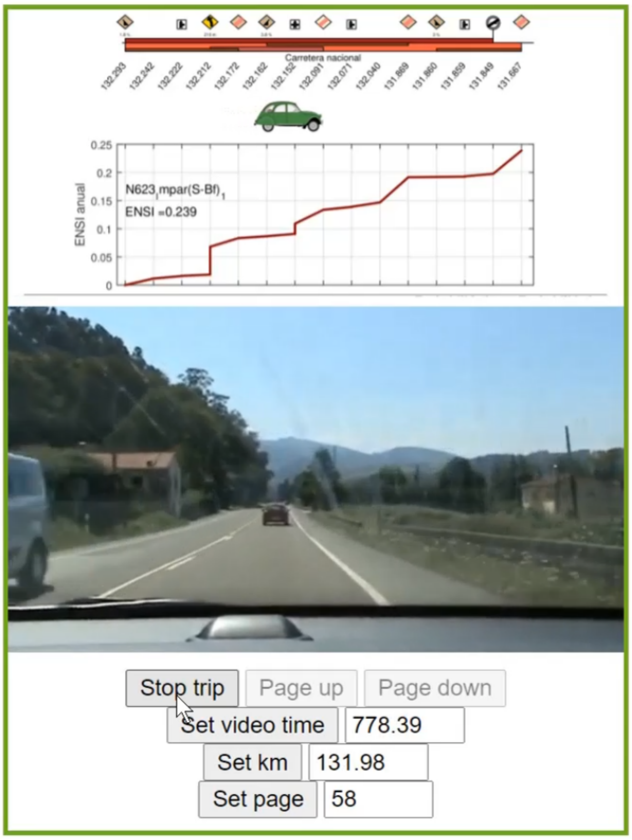


Figure 39: Graphic information provided by the application. Bayesian Network, items present on the road, along with their PKs, and accumulated risk graph, showing their jumps and slopes.

## 9 Contributions through YouTube Videos

My passion for teaching and the propagation of knowledge has led me to post more than 150 videos on YouTube, which fundamentally include the last doctoral courses that I taught before retiring, as well as other contributions with various dissemination and resolution videos. of useful problems for society.

## 10 New methods

The new methods that have appeared around artificial intelligence and Computer Science have shaken the traditional methods of Statistics, based on mathematical theory and demonstrations, with their positive and negative parts.

Today simulation techniques, with a serious theoretical justification, allow us to obtain, for example, very good approximations to the joint distributions of the variables or statistics that we want, without needing to have specific theoretical results, which allows us to substitute, for example , the confidence intervals or regions for the joint distributions, much more informative.

### 10.1 Testing of statistical hypotheses

Alternatives to classic problems have also appeared, such as statistical hypothesis testing, with the well-known bias in favor of null hypotheses.

As a posthumous tribute, I want to indicate that the book by Professor Javier Girón, RIP, which was carefully reviewed by his wife María Lina Martínez García, has added an alternative that overcomes these problems and gives them a very original, elegant and useful solution. I want to draw attention to the fact that the reception of their proposals has not received the support that I believe they deserve, despite being very necessary and relevant, see figure 40.

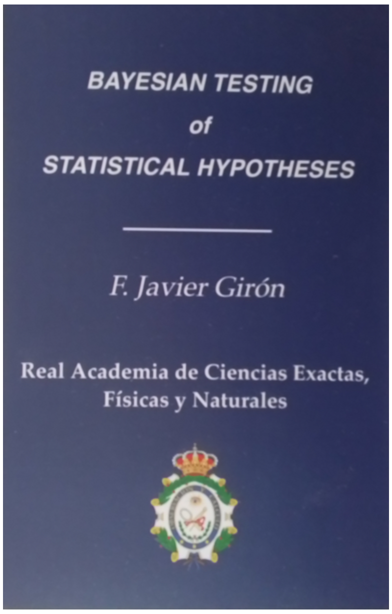


Figure 40: Book by Javier Girón on Contrast of statistical hypotheses.

### 10.2 Bayesian methods

Bayesian methods, especially combined with Bayesian networks, incorporating their parameters as new variables, lead to mixtures of distributions, which reproduce, in practice, any joint distribution and learn it from the data. Furthermore, these methods provide a natural and very powerful way of explaining the results, which other AI methods lack.

### 10.3 Artificial intelligence and ChatGPT

Our contributions also include artificial intelligence, see (Castillo et al., 1997), (Castillo et al., 2007) and (Castillo et al., 2015)).

The appearance of ChatGPT4 has represented the greatest revolution in the area of AI. Although it is true that these advances create important problems, such as their ugly or perverse use, which must be resolved, it is surprising to see how they are attacked and attempted to be banned, despite the fact that these advances and their expectations are spectacular for our society.

In fact, we are at a crucial moment for the future, in which it is important not to make mistakes in the decisions to be made, and in which the people with the greatest knowledge should govern the decision centers. Some principles that have been considered unalterable must be seriously questioned.

Finally, we must design a new way of teaching, much more open to creativity and free of paths and conditions, like those we have had for many years. A drastic change in content is necessary that breaks the mold, in the sense that it marks new possibilities that were previously unimaginable.

I suggest ChatGPT4 designers incorporate the idea of designing using properties and functional equations as a tool, as they would add a new and very powerful intelligent activity plus.

Figure 41: Aid to mothers in Benin and Togo.

## 11 Final thanks

I also want to indicate that the cash prize, following family tradition, has been donated to an institution in Togo that supports mothers. The idea is that they buy in the markets, wholesale at their level, and make profits daily. This allows them not only to repay the aid in less than a year so that other mothers can also benefit, but also to support themselves and their close relatives and accumulate enough capital to continue supporting themselves for life. The idea is as simple as “giving the fish and not the fish” and making it sustainable, see figure 41.

And finally, I would like to thank, in a very special way, my wife, a faithful and tireless companion throughout this journey, without whom I would not be here. Also to my children, who have lost their father many hours that he dedicated to this beautiful specialty of Statistics.

Thank you so much.

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