Posdem: frame and reliability

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Abstract

Try to reduce the sampling error for a specific survey population frame with the stratification and the sample size done is possible at any time. The sampling methods are tested over the frame with the purpose to make the sample more representative. POSDEM allows testing 28 different sampling methods over any population frame. We seek increased sample heterogeneity. Software is free and more sampling procedures can be added. It can be used in surveys, censuses, quality control or to explore frames into big data.

Sampling methods have a strong relationship with the survey population frame. Partly by this, there are many sampling methods that are not frequently applied. On the other hand we know that several alternatives of systematic sampling and unequal probabilities sampling are frequently used. However, both may be disturbed by the structure of the investigated population. Trends, cycles and inappropriate relationship between variables are present more often that it seems. In both cases, to find the best sampling method, it is necessary to study the relationship between the sampling plans and the population frame before applying these methods.

For different sample sizes, software POSDEM tests this relationship between frames and sampling methods. That test might be done just over the real population or over a super population model inspired on this population. We find interesting results regarding stability properties procedures

Keywords: Systematic sampling. Probabilities proportional to size. Superpopulation model.

AMS Classification: 62D05

Posdem: Marco y confianza

Resumen

Siempre es posible reducir el error de muestreo para un determinado marco poblacional sin variar el tamaño de muestra o la estratificación utilizada. Para conseguir que la muestra sea más representativa podemos estudiar los diferentes métodos de muestreo disponibles en relación con el marco utilizado. POSDEM permite contrastar 28 métodos sobre cualquier marco poblacional. Buscamos aumentar la heterogeneidad en la muestra. El programa está disponible en la web y puede ser utilizado en encuestas, censos, estudios de control de calidad o para explorar por muestreo marcos dentro de grandes poblaciones.

Los métodos de muestreo presentan una relación estrecha con su marco poblacional. Para diferentes tamaños de muestra, el software POSDEM, contrasta estas relaciones entre marcos y métodos de muestreo. Este contraste puede realizarse sobre el marco real o sobre el marco suprapoblacional basado en un modelo de dicho marco. De esta forma se ponen de manifiesto resultados relevantes acerca de la estabilidad de los procedimientos utilizados.

Observamos que se utilizan con frecuencia distintas alternativas de muestreo sistemático y muestreo con probabilidades desiguales. Sin embargo ambos pueden ser distorsionados por la estructura de la población investigada. Tendencias, ciclos o inapropiadas relaciones entre variables se presentan con mayor frecuencia de lo aconsejable. En todos los casos para elegir el mejor método de muestreo es necesario estudiar la relación entre el plan de muestreo y el marco poblacional antes de aplicar estos métodos.

Palabras clave: Muestreo sistemático. Probabilidades proporcionales al tamaño. Modelos de superpoblación.

Clasificación AMS: 62D05

1. Introduction

"Sampling is not mere substitution of a partial coverage for a total coverage. Sampling is the science and art of controlling and measuring the reliability of useful statistical information through the theory of probability" (Deming, W.E.).

Statistical projects have some characteristics in common. One of them is to be proud of the big sampling size we choose for our research. A study based in hundreds of thousands of sampling units is well considered. In some ways it is considered as a quality indicator. Sometimes it is true, but sometimes it is not. Doing things wrong many times is only a guarantee of being more deeply mistaken, of being in the bad direction. The sample units must have good quality for themselves. The optimum is a very small size: two units per stratum, which is how exactly we know the reliability of data. These situations in which we confuse a big sample size with quality are going to be even worse with the new analysis coming from big data. Perhaps we need to remember loudly the Mahalanobis work that shows early and clearly that, with sampling methods, a small group of interviewers trained properly obtained smaller errors than those in big operations (Camarero, L.).

Statistical information differs from other types of information in which that one must have added a measure of its reliability. Ideally, this measure accompanies the statistical data in the form of small parenthesis under the figure. The measure of reliability attached to the criterion, provided by the researcher, of validity is the one that configures what is called representiveness of statistical data in terms of probability theory under controlled uncertainty. A statistical definition of explaining these concepts can be seen in Sánchez-Crespo G; Manzano, V. at http://goo.gl/5gz9hX

Related with sampling plans POSDEM allows for 28 sampling methods answering among other questions: what is the best sampling plan for a specific population frame?; what is the sampling size related for a significance level?; what is the mean square error expected?; what are the units that must be investigated?; what happens if ...? (Biehler, R.).

The results obtained with the various available procedures of systematic sampling could depend on several factors such as the structure of the population and the hypothesis made (Bellhouse, D.R. and Rao, J.N.K.).

An example has been prepared in order to show the possibilities of this software. This paper examines several sampling methods to know what happens if in the structure of the population unexpected changes occur. For instance, it may be expected that population presents a linear trend but it actually has a polynomial trend, because there has been changes in the structure of the population since the last available information. Is there any sampling method robust enough when the population hypothesis made to apply the sampling methods have unexpected changes? With this software, this question can be answered. Another interesting feature is the behavior of the sampling methods when the degree of randomness changes.

Considering the observed erratic behavior, attention has been focused on the centered located systematic method, when the sample size increases as well as on the inconvenient need to differentiate if the sampling interval, k=N/n, is even or odd. Until now, only the Yates method of extreme corrections eliminated the linear trend for even or odd values of the sample size. The centered method of Madow does not eliminate the linear trend when the sampling interval, k, is even, and the balanced and modified systematic sampling procedures do not eliminate the linear trend when the number of units in the sample is odd.

2. Population and methods considered

The population presented by Murthy, M.N. (1967, p.127) and used by Krishnaiah, P.R. and Rao, C.R. (1988, p.131, population 4), will be applicable to illustrate the changes due to the fit of polynomial models with degrees one to five. The purpose is to check how the changes in the specification of the model, in the sample size or in the distribution of the random error, affect the mean squared error. The population considered consists of 128 units from the 1961 census.

The following methods have been analysed: random stratified sampling with one sample unit per stratum (0), systematic sampling with constant sampling interval (1), systematic sampling corrected in the extremes of Yates (2), balanced systematic sampling (3), modified systematic sampling (4), and centered systematic sampling with the constant interval of Madow (5) (). To these classical methods we add two new methods: systematic sampling with a variable sampling interval and its application to the centered method.

This method will be referred to as centered systematic sampling with a variable interval (6). We have been respectful to the numbers that Bellhouse and Rao gave to the methods.

Other methods, like probabilities proportional to size sampling methods (Sánchez-Crespo Rodriguez, J.L.), considered by POSDEM but not directly related to this paper, could be found in the following internet page: http://goo.gl/ZRkCw4

With POSDEM it is possible to connect these methods between them. Four basic methods have been considered: constant interval, variable, balanced and modified. And they have been to combine with the centered method and with the extreme corrections making a total of sixteen systematic methods.

The comparison of the systematic centered with a variable sampling interval, involving the methods of unequal probability sampling, will be left for a second article.

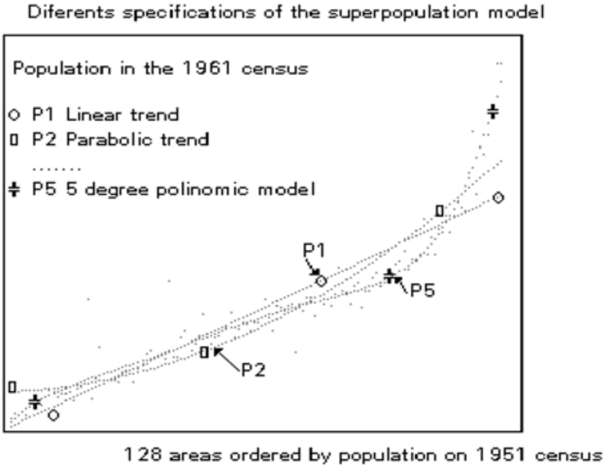
3. The superpopulation models

With the population considered the best fit corresponds to the polynomial model with five degrees (P5). Nevertheless it is also possible to suppose that the P1 and the P2 models represent the population with the purpose of seeing what would happen if the hypothesis that we made about the population changes.

With these models we are able to make a set of random populations that are in agreement with a patron. In each of these populations so generated it is possible to obtain all the possible samples and determine the mean square error of the estimator for different selection procedures. The set of the mean square error so calculated allows us to compute the expected value of the mean square error and its variance over the model.

Figure 1

Different models for the same population



4. Evaluation of sampling methods

To evaluate the sampling methods it is applied a superpopulation model adapted to each population. Some of the methods considered, centered and ends corrections, are biased so it is necessary to determine the mean squared error (mse).

$$mse\left(\hat{\hat{x}}\right) = E\left(\hat{\hat{x}} - \overline{X}\right)^2 \qquad [1]$$

POSDEM get the structure of the population fitting the data with the orthogonal polynomial method. Next the software generates populations defined by the model calculating the mse for each population and sampling method considered. The mean of the mse so calculated, over the set of populations generated, will be the approach to the expected value of the mse under the superpopulation model.

$$E^*\left(mse\left(\hat{\hat{x}}\right)\right) \cong \frac{\sum_{g=1}^{G} mse\left(\hat{\hat{x}}\right)_g}{G} \qquad [2]$$

Where g=1, 2...G represents the set of finite populations generated with the model.

This evaluation procedure assures the link of this work with the work of Bellhouse, D.R. y Rao, J. N. K. The results obtained for models of degrees one and two are coincident and let us suppose that for higher degrees it will also be coincident. The symbols used in the results tables will be E*(mse (number)), where E* is the expected value over the model and number represents the sampling method considered.

To know in which way POSDEM allows evaluate sampling methods it is necessary to see the next part call mean squared error stability: A new upper confidence limit to evaluate sampling methods.

5 Mean squared error stability: upper confidence limit to evaluate sampling methods

The variance over the model is considered the measure of the accuracy for the mean squared error. And it is done by the following expression:

$$V^*\left(mse\left(\hat{\hat{x}}\right)\right) \cong \frac{\sum_{g=1}^{G}\left(mse\left(\hat{\hat{x}}\right)_g - E^*\left(mse\left(\hat{\hat{x}}\right)\right)\right)^2}{G} \qquad [3]$$

It could be observed that if the random error of the model increases then the centered method would become erratic, because the variance over the model increases too. This is the reason to evaluate the sampling methods using an upper confidence limit trough the expected mse value and his deviation. The expression results +1.96 allows to have a confidence limit about that the expected error will not be greater than that this upper confidence limit, at least in 95% of 100 populations generated with this model. The sampling method will be selected considering the expected value of the mse and his variance over the superpopulation model.

6 The original population and its relation to the superpopulation approach

This table is introduced with the aim of showing that the calculations carried out by the computer program POSDEM are done correctly.

Table 1

The mean square error for different methods in the original population presented by Krishnaiah, P.R. and Rao, C.R. (1988, p.131, population 4)

	n=4	*n=8*	*n=16*	*n=32*
Systematic constant interval (1)	487,706.63	133,189.25	39,648.80	5,884.60
Centered with const.inter.	66,012.53	2,330.25	8,664.66	94.76
Extreme corrections	69,135.45	19,228.90	3,401.43	1,152.19

Table 2

Shows the relationship between the mean square error computed in the natural population, the expected value taken over the model and the variance of the mean square error over the model. If a selection procedure is chosen regarding the result obtained in columns (1), (2) or (4) we have columns (5) to (7) that show the preferable method for each hypothesis. The column (4) is a confidence limit of the mean square error over the model

	(1)	*(2)*	*(3)*	*(4)*	*Selection procedures choose using*		
σ_e =200	*mse*	*E*(mse)*	*V*(mse)*	$CL^{*}=E^{*}(mse)+2^{*}\sqrt{V^{*}(mse)}$	*mse*	*E**	*CL**
Cwci_n=4	66,012.53	21,624.19	313,842,251.65	57,055.38	Cwci	Cwci	Cwci
Ec_n=4	69,135.45	47,383.00	42,475,069.38	60,417.58			
Cwci_n=8	2,330.25	6,781.00	38,985,713.00	19,268.71	Cwci	Cwci	
Ec_n=8	19,228.90	11,213.00	8,841,197.00	17,159.83			Ec
Cwci_n=16	8,664.66	3,807.60	11,919,358.68	10,712.48			
Ec_n=16	3,401.43	2,812.13	2,145,672.35	5,741.75	Ec	Ec	Ec
Cwci_n=32	94.76	2,232.56	4,461,746.87	6,457.13	Cwci		
Ec_n=32	1,152.19	969.48	571,906.83	2,481.97		Ec	Ec

In the example where n=8 the expected value of Centered with Constant interval is clearly better than the Extreme Corrections. Nevertheless, the variance over the model is considerably different for these methods and when choosing them with the proposed Upper Confidence Limit the results become better for the Extreme Corrections.

7. Populations structures under the superpopulation models with POSDEM

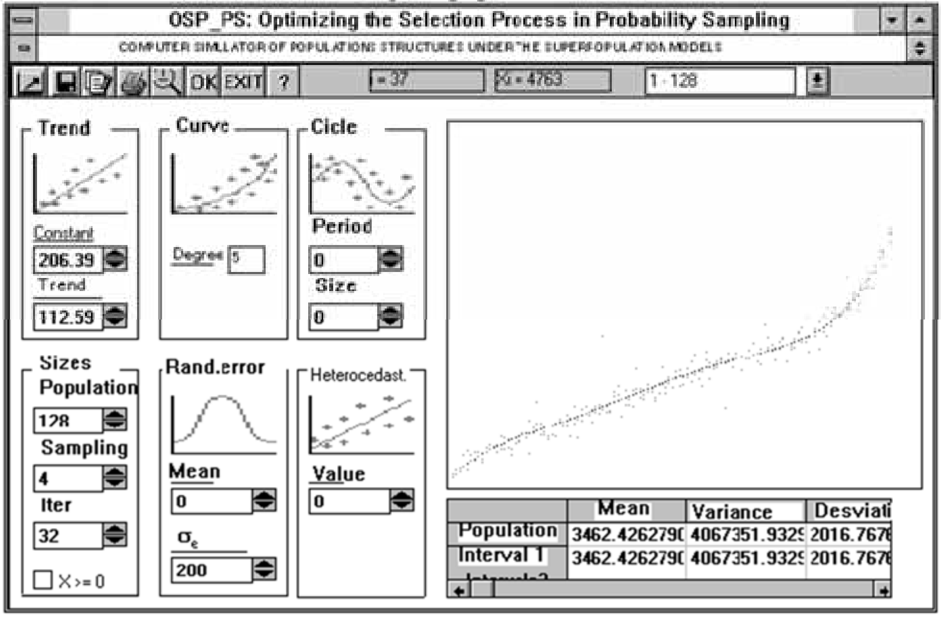
Here the module of simulation of structures is used in the application POSDEM. This makes it possible to define a superpopulation model, which takes into consideration the population used, with the expression:

$$X_u = a_0 + a_1\ U1 + a_2\ U2 + a_3\ U3 + a_4\ U4 + a_5\ U5 + e_u \qquad [4]$$

Where **U** represents the population units that in this case takes values between 1 and 128. $\mathbf{a_i}$ with i=1, 2, 3, 4 and 5 are the parameters computed by the least squared method. $\mathbf{e_u}$ is a random error term where $E^*(e_u)=0$; $E^*(e_u\,e_v)=0$ with $u\neq v$; $E^*(e_u^2)=\sigma^2_e$, E* represents the expected values in respect to the model.

Figure 2

POSDEM screen to modelized original populations



OSP_PS: Optimizing the Selection Process in Probability Sampling

COMPUTER SIMULATOR OF POPULATIONS STRUCTURES UNDER THE SUPERPOPULATION MODELS

i = 37 | $\bar{x}_i$ = 4763 | 1 - 128

Trend: Constant 206.39; Trend 112.59

Curve: Degree 5

Cicle: Period 0; Size 0

Sizes: Population 128; Sampling 4; Iter 32; □ X>=0

Rand.error: Mean 0; σ_e 200

Heterocedast.: Value 0

	Mean	Variance	Desviati
Population	3462.4262790	4067351.932	2016.767
Interval 1	3462.4262790	4067351.932	2016.767

Table 3

The table below shows the estimators obtained for different degrees of the parabolic model used to represent the population showed by Murthy, M.N. (1967, p127)

Models	a_0	a_1	a_2	a_3	a_4	a_5
P1	149.08	51.38				
P2	1,179.16	-3.84	0.368			
P3	-83.03	119.02	-1.854	0.0115		
P4	728.48	-2.54	2.347	-0.0390	-0.0002	
P5	206.39	112.59	-3.803	0.0873	-0.0009	-0.000003

The results obtained when this model, using the random generation of populations, must be in agreement with the following theoretical results provides by Bellhouse, D.R. and Rao, J.N.K:

A) For the linear case:

1) Systematic sampling: $E^*V_p\left(\hat{\bar{x}}_{sis}\right)=a_1^2\left(k^2-1\right)\big/12+\bar{\sigma}^2$ with $\bar{\sigma}^2=\sigma_e^2\left(k-1\right)/nk$

The first variance component of the estimator is due to the linear trend, and the second to the random error.

2) Random sampling: $E^{*}V_{p}\left(\hat{\bar{x}}_{sr}\right)=a_{1}^{2}(k-1)(nk+1)/12+\bar{\sigma}^{2}$

3) Stratified sampling with one unit for stratum: a population with n strata formed by the next sets, is used: $\{1...k\}\{k+1,...2k\},...\{(n-1)k+1,...nk\}$. $E^{*}V_{p}\left(\hat{\bar{x}}_{str}\right)=a_{1}^{2}\left(k^{2}-1\right)/12n+\bar{\sigma}^{2}$

It is easy to see that in this case: $E^{*}V_{p}\left(\hat{\bar{x}}_{strat}\right)\leq E^{*}V_{p}\left(\hat{\bar{x}}_{sis}\right)\leq E^{*}V_{p}\left(\hat{\bar{x}}_{sr}\right)$

B) For the non linear case:

The following formula is used to check results when the degree of the polynomial model is two. E*(mse(2)) – E*(mse(5))= $(a_2^2/720)(k^2-1)(19k^2-31)>0$ for k odd

The mean square error is used because the centered (method 5) and corrected (method 2) procedures are biased methods.

In conclusion, with small samples (5) is better than (2) and this trend is reversed for large samples. When there is doubt between the application of (5) or (2), method (6) should be used since it is more stable than (5) and (2) in relation to changes in the sampling size.

8. Conclusions

The main results of this work are:

1. The implementation of a computer program to carry out the optimisation of the sampling selection and to evaluate alternative designs for specific populations under the superpopulation approach. This program could be useful for students studying the theory and practice of sampling in finite populations, in both their basic and advanced versions; for professors that want to provide an instrument for the empirical investigation over the area of sampling surveys; and for central statistical offices, companies, or researchers. This new program is currently available for use on the Internet. http://goo.gl/LjHGm1

2. The results obtained using these sampling methods on various populations, are in agreement with the theory. However, we found an interesting result: sampling methods, which are excellent in some situations, are very bad in others. The new systematic sampling method proposed, centered with variable interval, behaves well in different situations. In various examples, it reduces the mean square error by comparison to other systematic procedures considered. This reduction is due strongly to the variations of the sampling size, the numbers of groups in the population, or if they are even or odd. This method also works when, in the population, a structure change occurs. This avoids the risk of choosing a sampling method which behaves badly if the population has unexpected changes.

3. In view of the experimental increases in the random nature of the population, the centered methods presents a small expected error, as the theory says, but with a large variability, as practice shows. This suggests a new indicator to measure the sampling error

to be used to improve the evaluation of systematic sampling methods. This indicator is an upper confidence limit of the mean square error distribution over the superpopulation model. So it is possible to explain and control the erratic behavior observed in the centered systematic sampling in relation to the random term of the considered model.

Reduce the sampling error for a specific survey population frame with the stratification and the sample size done it is possible if we increase the sample heterogeneity. Sampling methods have a strong relationship with the survey population frame. POSDEM test sampling and frames to find best sample that mean less error limits.

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